# Mécanique de la rupture

## … et son lien avec la métallurgie

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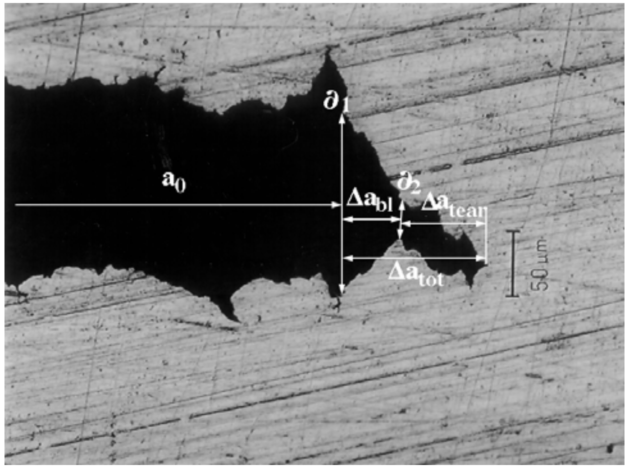


**Action Nationale de Formation (ANF), METALLURGIE FONDAMENTALE**
**22 - 25 Octobre 2012, Aussois, France**

UCL Université catholique de Louvain

# Plan de la présentation

**1. Introduction**
Pourquoi la mécanique de la rupture ?

**2. Bases de la mécanique linéaire élastique de la rupture**
Notions de champs en K, G, taille de zone plastique $K_{IC}$, $G_{IC}$, validité

**3. Bases de la mécanique élastoplastique de la rupture**
Notions de champs de HRR, J, CTOD, JR curve, grandes déformations

**4. Lien entre la ténacité et la microstructure/mécanismes dans les métaux**
A. Rupture fragile – modèle RKR
B. Rupture ductile – modèle de croissance-coalescence de cavités
C. Cas des tôles minces métalliques

**5. Limites de la mécanique de la rupture**
Effets de géométrie, paramètre unique, invalidité, complémentarité avec approche par micromécanique de l’endommagement

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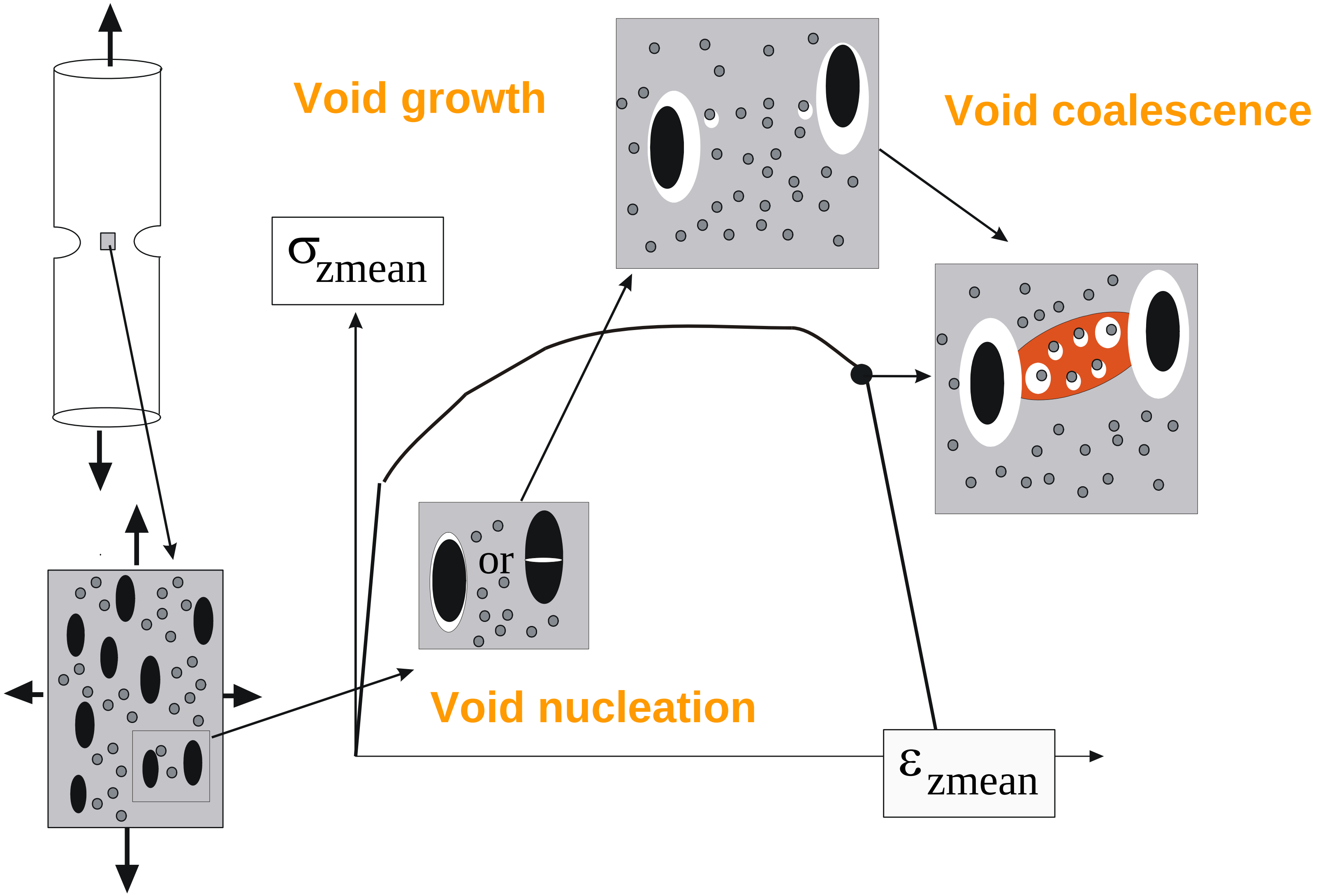
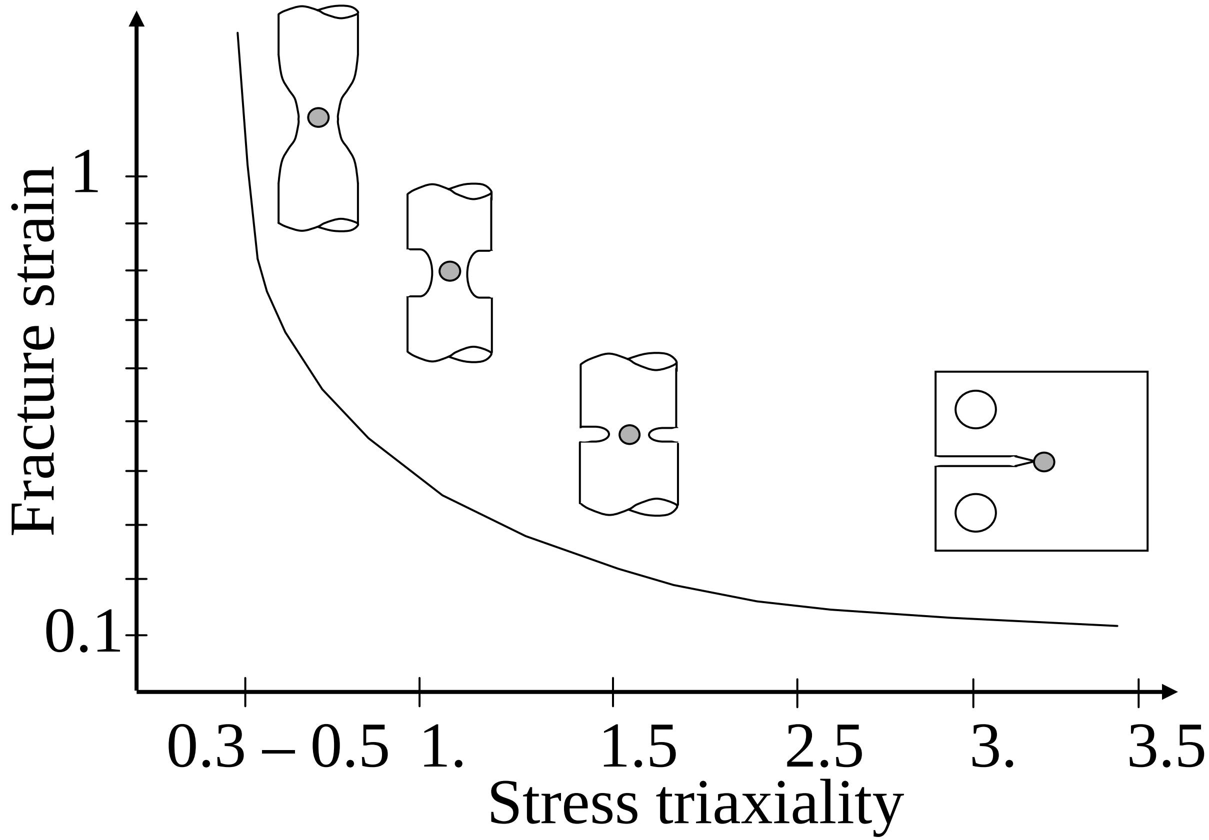
# Ductile fracture of metals

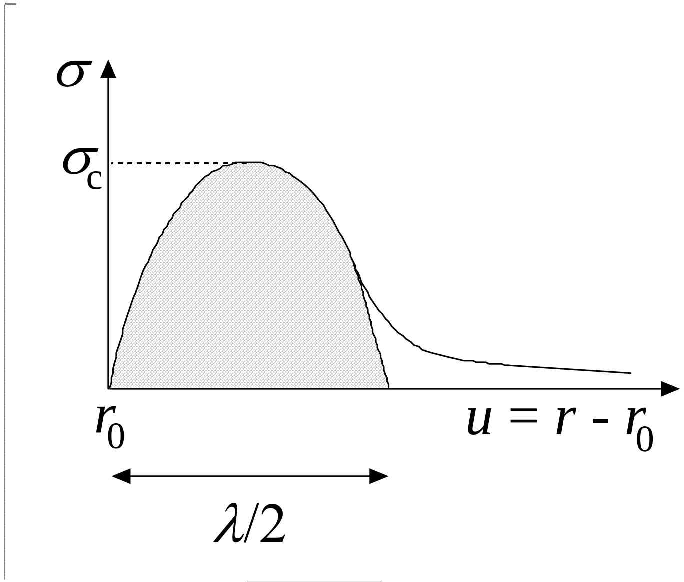


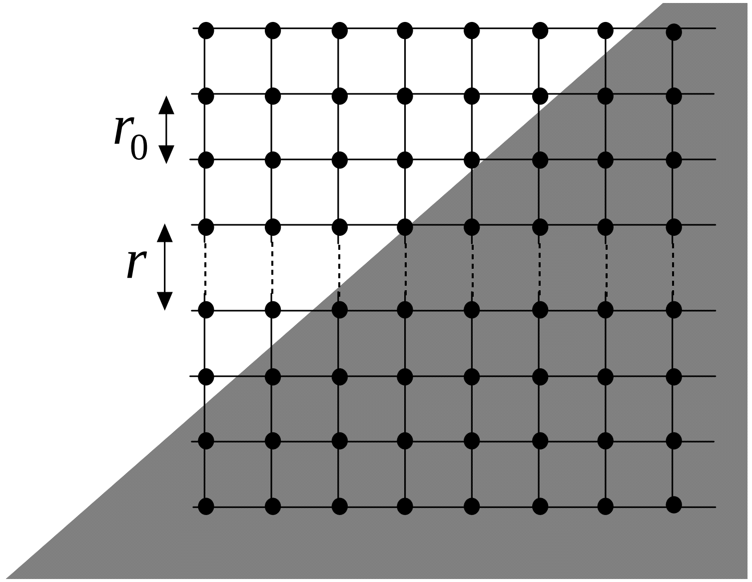
# The fracture strain (and fracture stress) is not a material property




*Can we predict fracture strain from damage mechanisms ? Yes, but this is another (very interesting) story !*

# Theoretical cleavage stress (brittle fracture)




$$\sigma_c = \sqrt{\frac{E\gamma_s}{r_0}}$$

$$\gamma_s \approx E r_0/20$$

$$\sigma_c/E \cong 1/5$$

**Experimental fracture stress at least one order of magnitude smaller !**

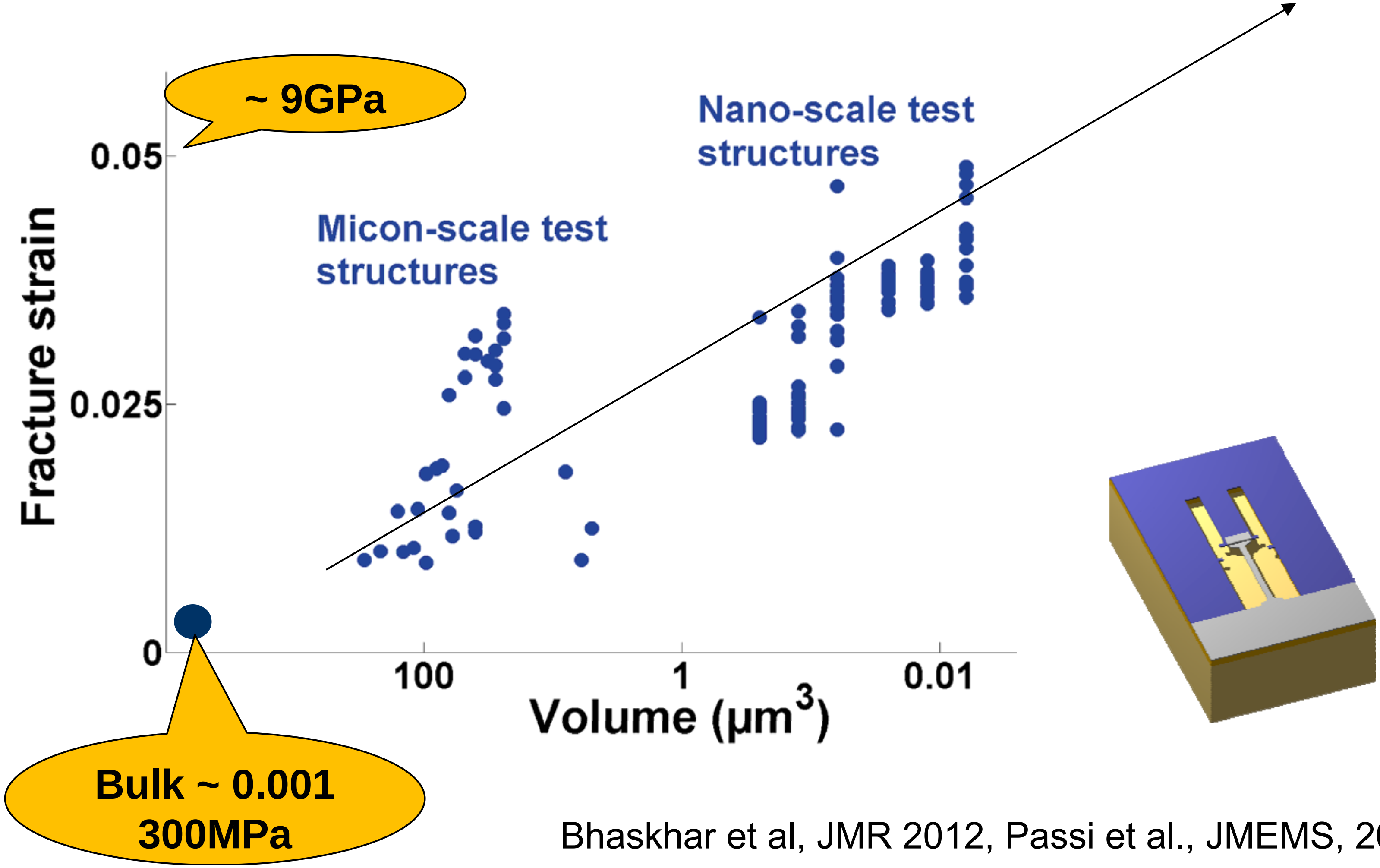
**Why ?**

**Because of the presence of defects**

# The fracture stress (or strain) is not a material property for brittle fracture

**Example for Si**

**Theoretical value ~ 20GPa**




Bhaskhar et al, JMR 2012, Passi et al., JMEMS, 2012

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# Ideal strength of silicon: An *ab initio* study

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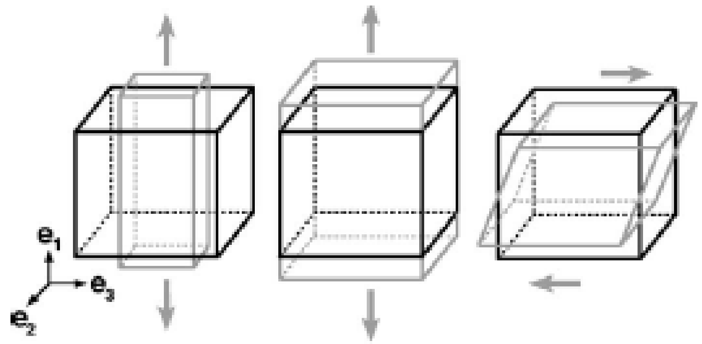
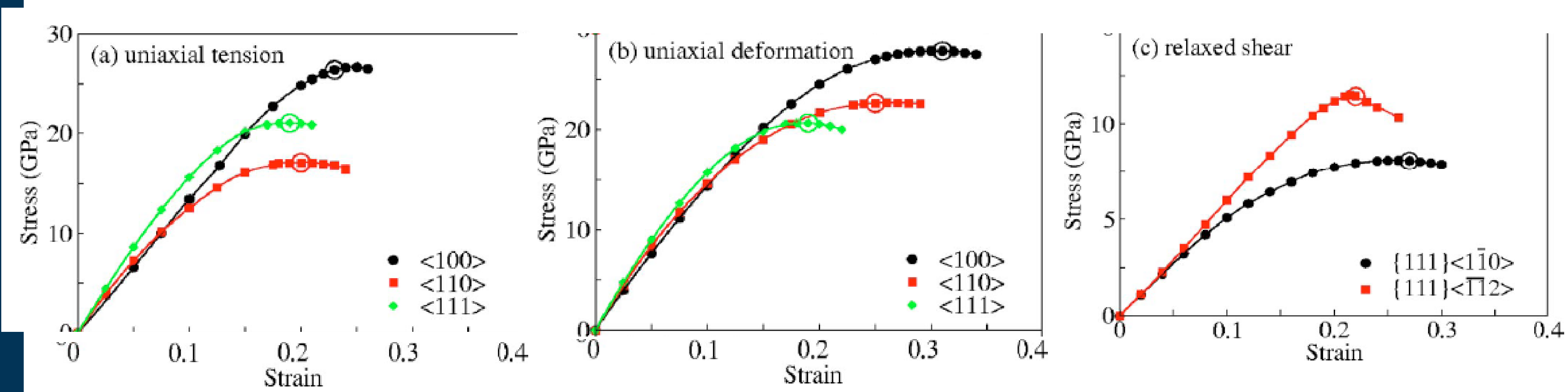
**Note also dependence of fracture stress on orientation and loading conditions**



# Fracture mechanics in a nutshell

- **Start with a sharp initial crack**

*« the worst possible mechanical defect »*

- **What is the load/energy needed for cracking initiation (beginning of propagation) – the resistive force ?**

*Define and determine an « as intrinsic as possible » fracture toughness property*

- **What is the load/energy required to pursue the crack propagation ?**

*Define and determine an « as intrinsic as possible » tearing resistance property*

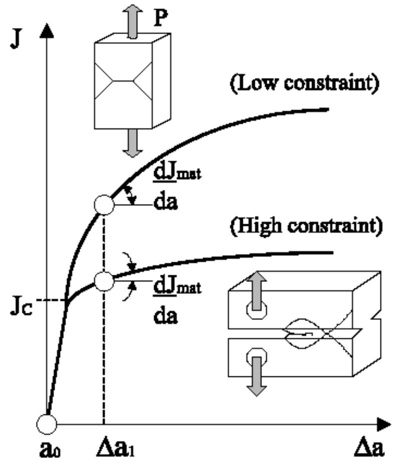
- **What is the load/energy available – the driving force ?**

- **Compare driving force and resistive force to assess the integrity of a structure – question of transferability**

# Applications

## 1. Structural integrity community

*Transferability from laboratory specimens to real structures (pipelines, nuclear power plants, airplanes,… ) – crack stability analysis*

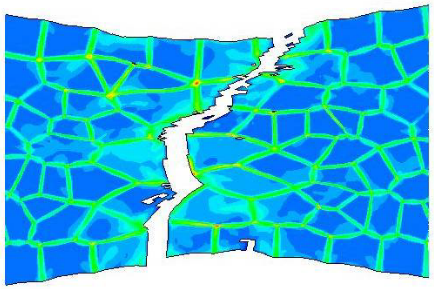



## 2. Metal forming community

*Some problems of forming are dominated by ductile tearing resistance*

## 3. Material scientists (all families)

*Quantify fracture toughness to compare materials + link with microstructure*



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**Let's talk first about the driving force**

## 5. Limites de la mécanique de la rupture
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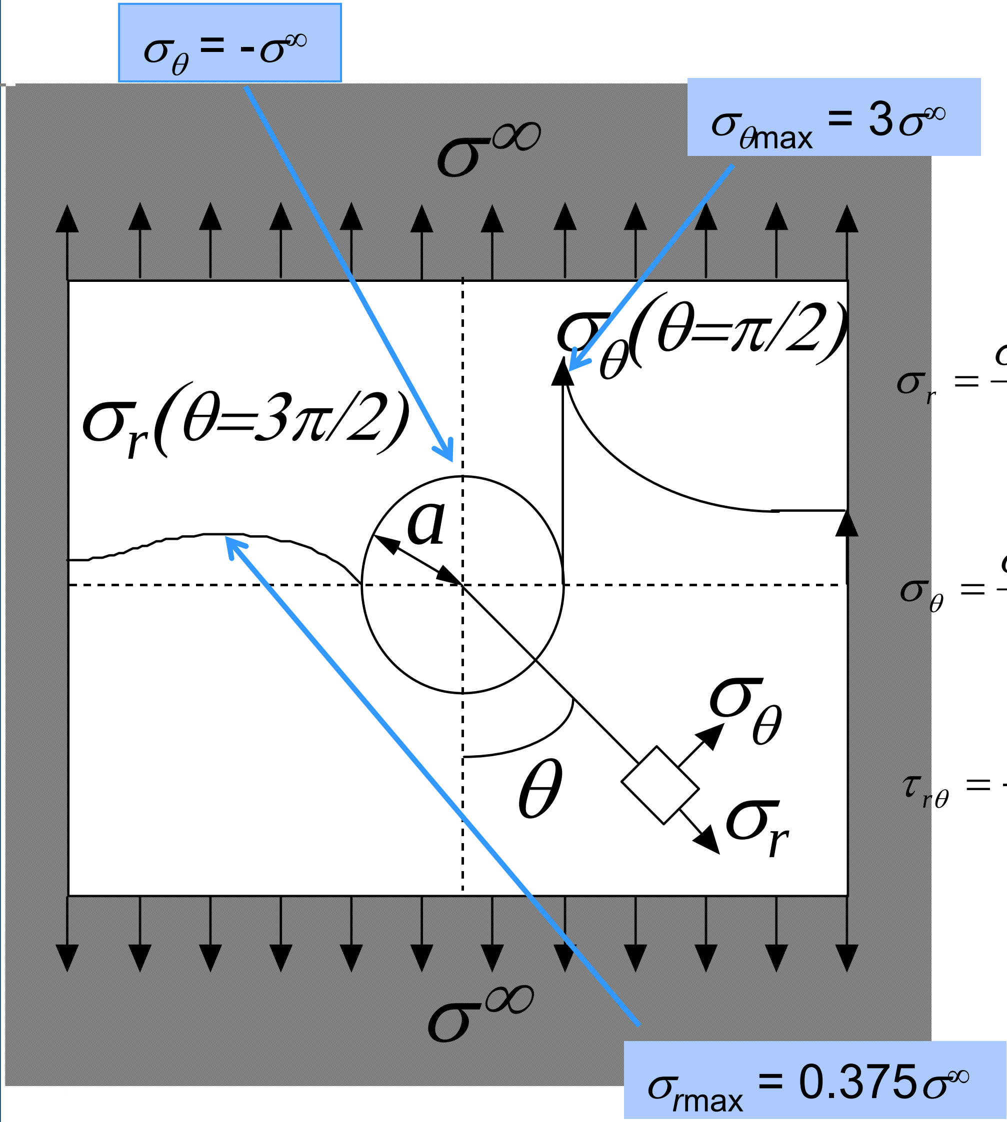
# Starting point : stress concentration around a hole in a plate



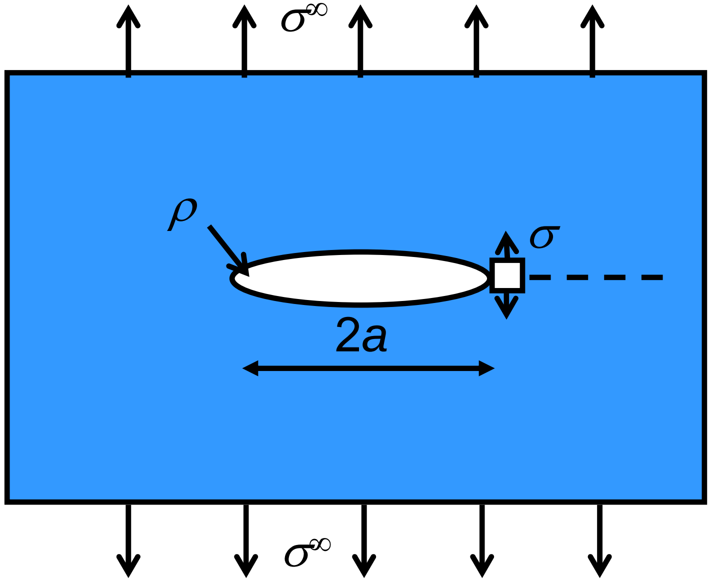

$$\sigma_r = \frac{\sigma^\infty}{2}\left(1-\frac{a^2}{r^2}\right) + \frac{\sigma^\infty}{2}\left(1+3\frac{a^4}{r^4}-4\frac{a^2}{r^2}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{\sigma^\infty}{2}\left(1+\frac{a^2}{r^2}\right) - \frac{\sigma^\infty}{2}\left(1+3\frac{a^4}{r^4}\right)\cos 2\theta$$

$$\tau_{r\theta} = -\frac{\sigma^\infty}{2}\left(1-3\frac{a^4}{r^4}+2\frac{a^2}{r^2}\right)\sin 2\theta$$

$$k_t = \frac{\sigma_{max}}{\sigma^\infty} = 3$$

# Elliptical hole and crack

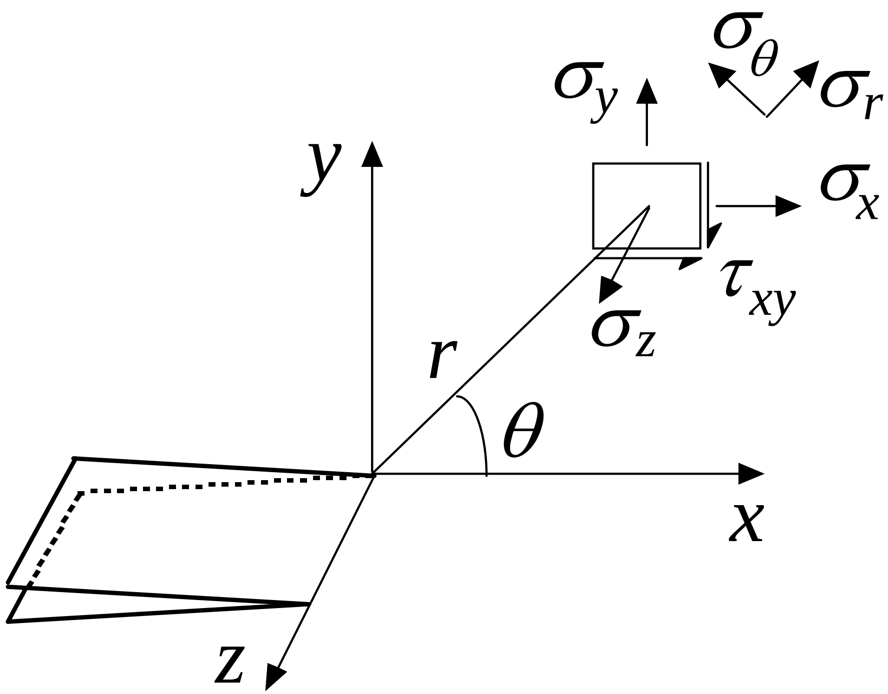



$$\sigma_{\max} = \sigma^{\infty}\left(1 + 2\sqrt{\frac{a}{\rho}}\right)$$

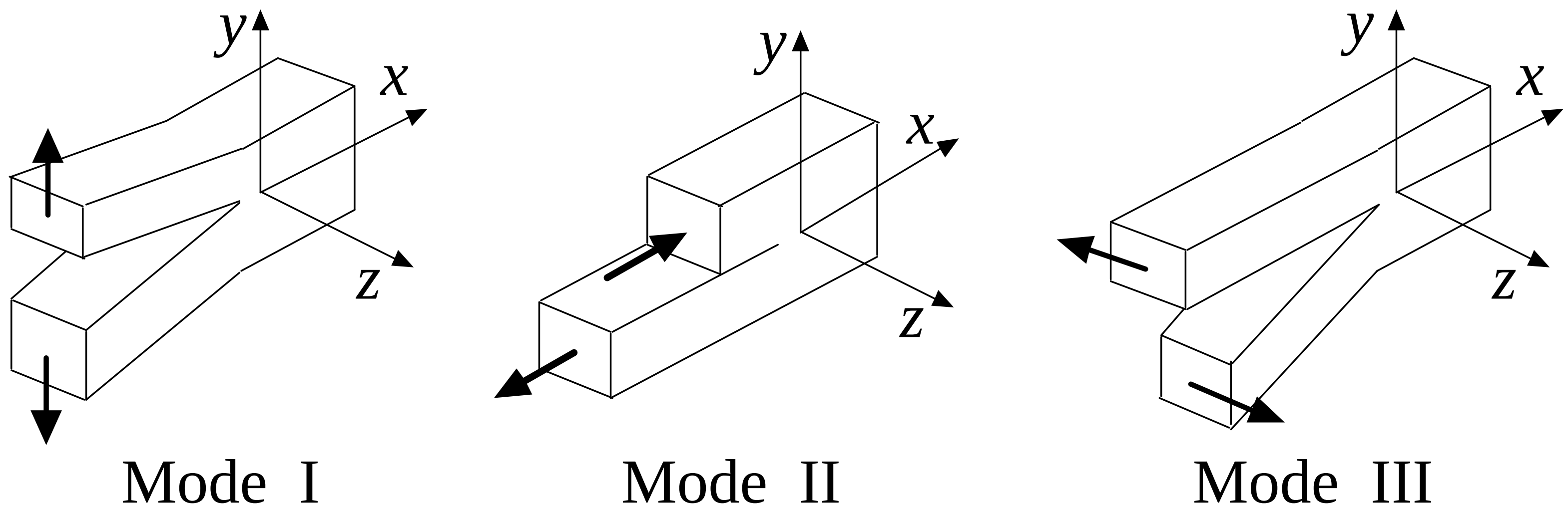
$\rho \to 0 \Rightarrow$ “stress singularity”

Only for pure ideal elastic behaviour :
“Linear Elastic Fracture Mechanics” (LEFM)

# Definition of coordinate axes for the stress-strain field around a crack tip



# Three different modes of crack opening



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# Assume isotropic linear elasticity

$$\varepsilon_x^{el} = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_z^{el} = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\varepsilon_y^{el} = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right]$$

$$\gamma_{yz}^{el} = \frac{1}{\mu}\tau_{yz}$$

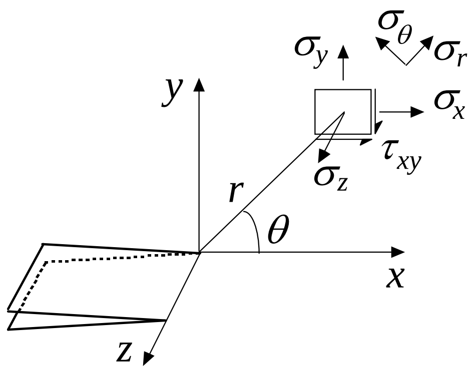
$$\gamma_{zx}^{el} = \frac{1}{\mu}\tau_{zx}$$

$$\gamma_{yx}^{el} = \frac{1}{\mu}\tau_{yx}$$

$$\boxed{\varepsilon_{ij}^{el} = \frac{1+\nu}{E}\sigma_{ij} - 3\frac{\nu}{E}\sigma_{\mathrm{m}}\delta_{ij}}$$

$$\mu = \frac{E}{2(1+\nu)}$$

# Stresses around crack tip can be expressed as series expansion




*a* = “length” of the crack

$$\sigma_{ij}^{\mathrm{I}} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} f_{1ij}^{\mathrm{I}}(\theta) + \sum_{n=2,3,\ldots}^{\infty} C_n^{\mathrm{I}}\left(K_{\mathrm{I}}, a, \text{other dimensions}\right) r^{\frac{n}{2}-1} f_{nij}^{\mathrm{I}}(\theta)$$

$$\sigma_{ij}^{\mathrm{II}} = \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}} f_{1ij}^{\mathrm{II}}(\theta) + \sum_{n=3,4,\ldots}^{\infty} C_n^{\mathrm{II}}\left(K_{\mathrm{II}}, a, \text{other dimensions}\right) r^{\frac{n}{2}} f_{nij}^{\mathrm{II}}(\theta)$$

$$\tau_{iz}^{\mathrm{III}} = \frac{K_{\mathrm{III}}}{\sqrt{2\pi r}} f_{1i}^{\mathrm{III}}(\theta) + \sum_{n=3,5,\ldots}^{\infty} C_n^{\mathrm{III}}\left(K_{\mathrm{III}}, a, \text{other dimensions}\right) r^{\frac{n}{2}-1} f_{ni}^{\mathrm{III}}(\theta)$$

The first term is the asymptotic solution when $r \to 0$

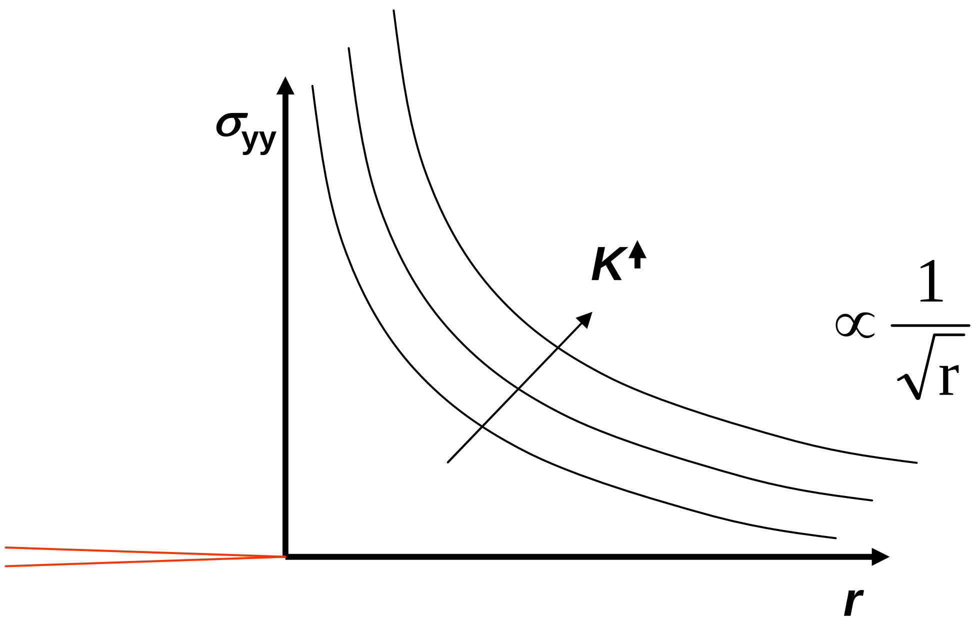
# Asymptotic stress field (in mode I)

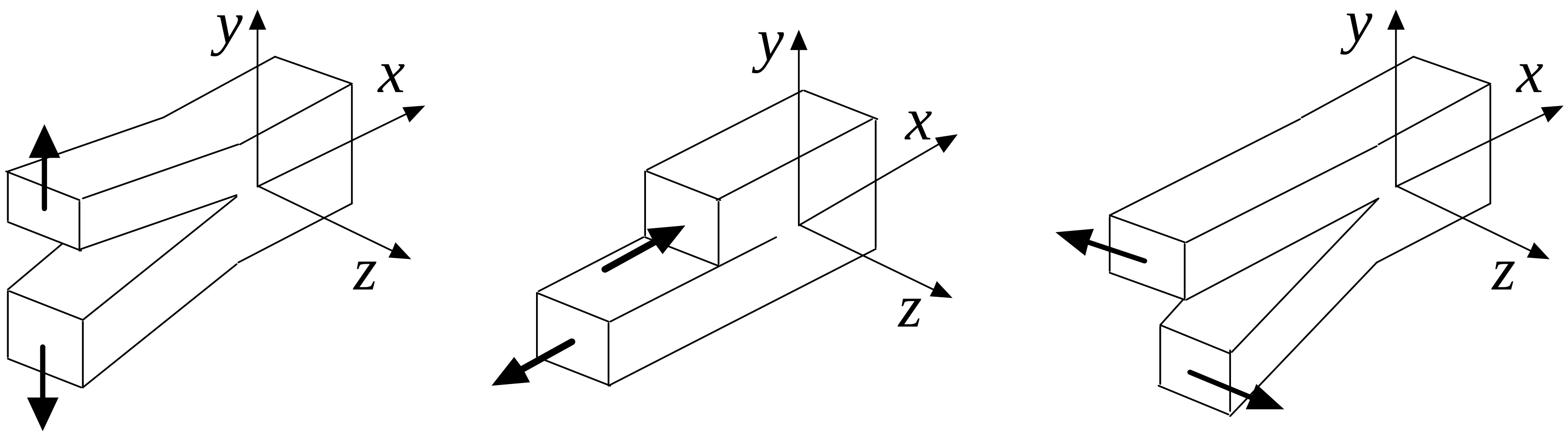
$$\sigma_{ij}^{\mathrm{I}} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} f_{1ij}^{\mathrm{I}}(\theta)$$

Plane strain (thick plate) $\sigma_z = \nu(\sigma_x + \sigma_y)$

Plane stress (thin plate) $\sigma_z = 0$

$K(a, \sigma^\infty, \text{geometry})$ = "Stress Intensity Factor" ($MPa.m^{1/2}$)






Mode I　　Mode II　　Mode III

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \qquad \sigma_x = -\frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \qquad \sigma_x = \sigma_y = \sigma_z = 0$$

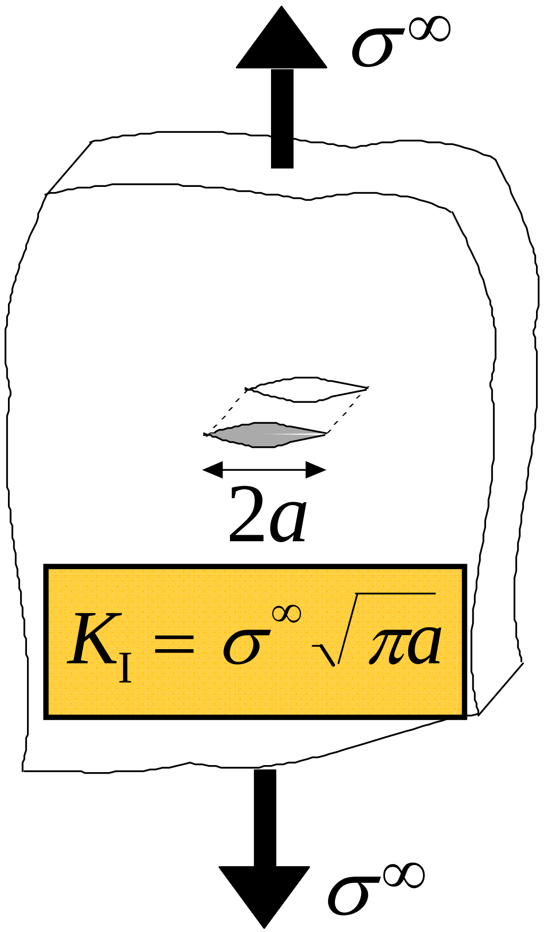
$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \qquad \sigma_y = \frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \qquad \tau_{xz} = -\frac{K_{III}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} \qquad \tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \qquad \tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}$$

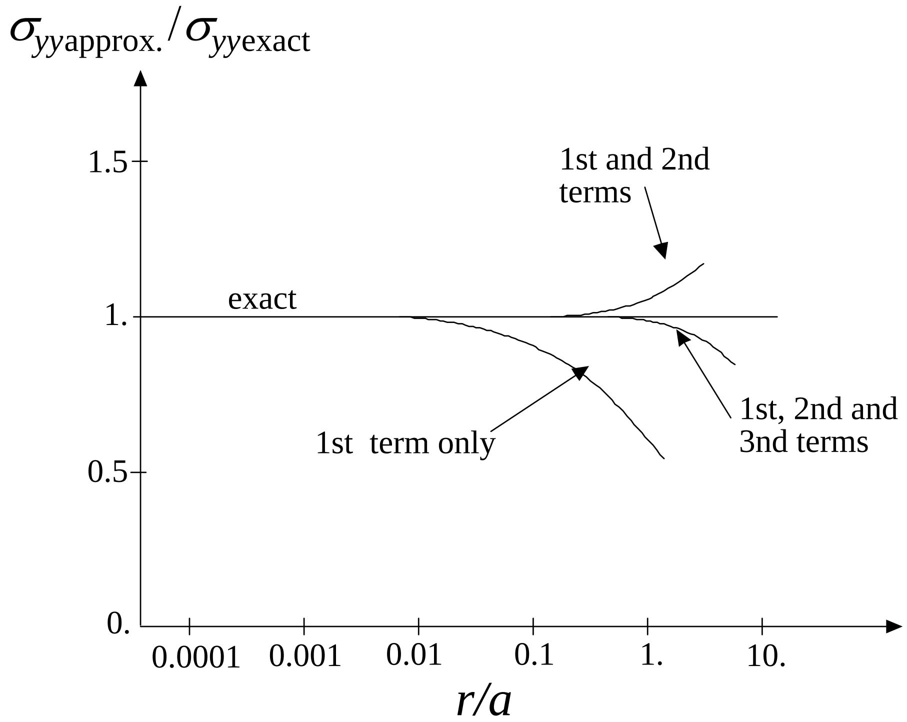
**General solution**

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{1ij}^{I}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{1ij}^{II}(\theta) + \frac{K_{III}}{\sqrt{2\pi r}} f_{1ij}^{III}(\theta)$$

# Example : through crack in a plate




$$K_I = \sigma^\infty \sqrt{\pi a}$$




In general $$K_{\text{I or II or III}} = Y\sigma^\infty \sqrt{a}$$

*Y* = non-dimensional factor that depends only on the geometry of the specimen

# Singular term for displacements $u_i$ (mode I)

$$u_x = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(k-1+2\sin^2\frac{\theta}{2}\right) \qquad u_y = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(k+1-2\cos^2\frac{\theta}{2}\right)$$

with $k = 3 - 4\nu$ in plane strain and

$$k = \frac{3-\nu}{1+\nu}$$ in plane stress

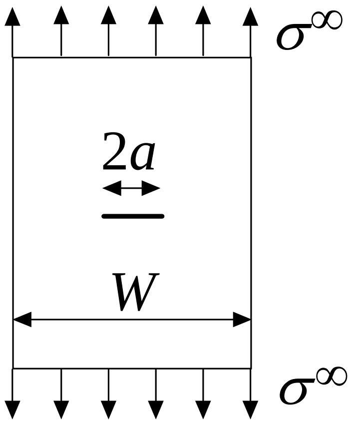
Displacement of the faces of the crack (“crack opening displacement”)

$\theta = \pi$

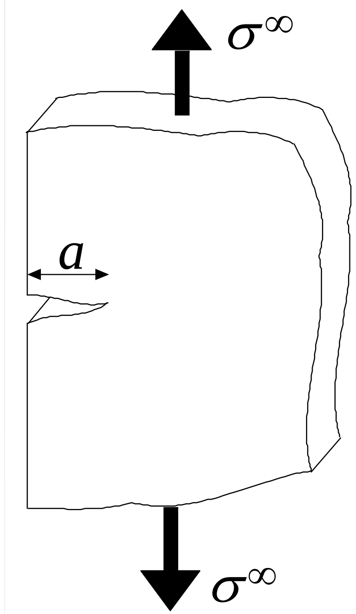
$$\delta = u_y(r,\pi) - u_y(r,-\pi) = \frac{8K_I}{E^*}\sqrt{\frac{r}{2\pi}}$$

with $E^* = E$ in plane stress and $E^* = E/(1-\nu^2)$ in plane strain

# Examples of other typical $K$ expressions




$$K_I = \sqrt{\pi \sec\left(\frac{\pi a}{W}\right)}\, \sigma^{\infty} \sqrt{a}$$




$$K_I = 1.12\sqrt{\pi}\, \sigma^{\infty} \sqrt{a}$$

See handbooks or use FE to generate new solutions

# “Strain energy release rate” $\mathcal{G}$

$$\Delta W_e - F\Delta u = \Delta \mathcal{P} = -\mathcal{G}\Delta A$$

$$\mathcal{G} = -\frac{\partial \mathcal{P}}{\partial A} \left(\mathrm{Jm}^{-2} \ or \ \mathrm{Nm}^{-1}\right)$$

$\mathcal{G}$ does not dependent of the mode of loading (see demonstration next)

$$\mathcal{G} = \int_0^u \left(\frac{\partial F}{\partial A}\right)_u du = -\int_0^F \left(\frac{\partial u}{\partial A}\right)_F dF$$

$\mathcal{G}$ = “driving force for crack extension” or “crack extension force”

# Relation between $\mathcal{G}$ and compliance C

**Crack extension at constant load**

$$\Delta W_e - F\Delta u = \Delta \mathcal{P} = -\mathcal{G}\Delta A$$

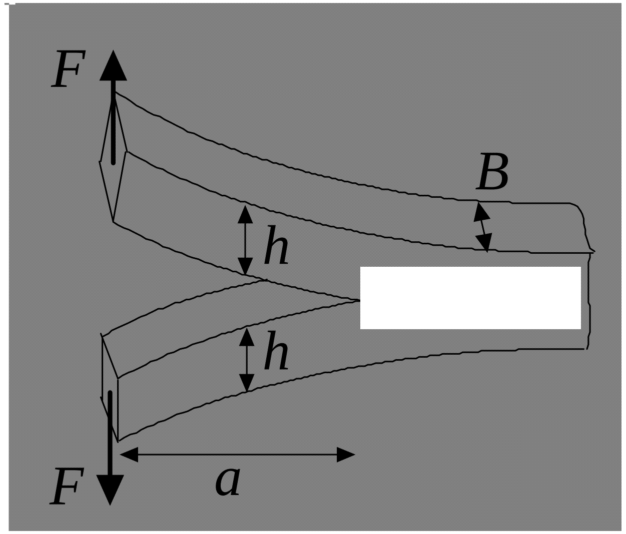
$$\Delta W_e - F\Delta u = \frac{Fu}{2} - Fu = -\frac{Fu}{2}$$

$$\mathcal{G} = -\frac{\partial \mathcal{P}}{\partial A} = -\left(\frac{\partial(-Fu/2)}{\partial A}\right)_F = \frac{F^2}{2}\left(\frac{\partial}{\partial A}\left(\frac{u}{F}\right)\right)_F = \boxed{\frac{F^2}{2}\frac{\partial C}{\partial A}}$$

**Crack extension at constant displacement**

$$\mathcal{G} = -\frac{\partial \mathcal{P}}{\partial A} = -\frac{u}{2}\left(\frac{\partial F}{\partial A}\right)_u = -\frac{u^2}{2}\left(\frac{\partial(F/u)}{\partial A}\right)_u = -\frac{u^2}{2}\left(\frac{\partial(1/C)}{\partial A}\right)_u = \frac{u^2}{2C^2}\frac{\partial C}{\partial A} = \boxed{\frac{F^2}{2}\frac{\partial C}{\partial A}}$$

# Example : "double cantilever beam"

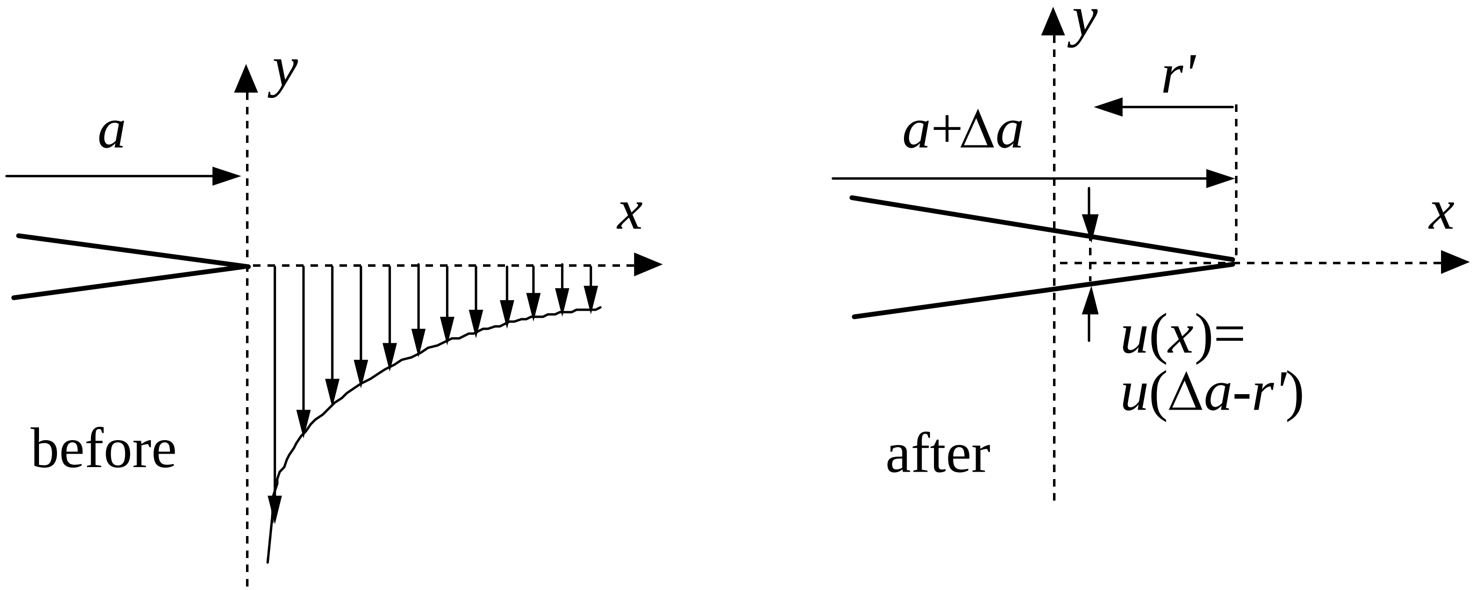



Assumption : the two beams are built in (encastered) at the position corresponding to the crack tip (this is not exact … this is an assumption)

$$\frac{u}{F} = \frac{8A^3}{EB^4h^3} = \frac{8a^3}{EBh^3}$$

$$\mathcal{G} = \frac{12F^2A^2}{EB^4h^3} = \frac{12F^2a^2}{EB^2h^3}$$

# $\mathcal{G}$ is univocally related to *K*




$$\Delta W_{\text{closing}} = 2B\int_0^{\Delta a} \frac{1}{2}\sigma_y(x)u_y(x)dx$$

# $\mathcal{G}$ is univocally related to *K*

$$\Delta W_{\text{closing}} = 2B\int_0^{\Delta a}\frac{1}{2}\sigma_y(x)u_y(x)dx$$

$$u_y = \frac{K_\text{I}}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(k+1-2\cos^2\frac{\theta}{2}\right)$$

$$\sigma_y = \frac{K_\text{I}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\Delta W_{\text{closing}} = B\int_0^{\Delta a}\frac{K_\text{I}(a)}{\sqrt{2\pi x}}\frac{K_\text{I}(a+\Delta a)}{2\mu}\sqrt{\frac{\Delta a - x}{2\pi}}(k+1)dx$$

$$= \frac{BK_\text{I}(a)K_\text{I}(a+\Delta a)(k+1)}{4\pi\mu}\int_0^{\Delta a}\sqrt{\frac{\Delta a - x}{x}}dx$$

Plane stress $$\Delta W_{\text{closing}} = \frac{BK_\text{I}(a)K_\text{I}(a+\Delta a)\Delta a}{E}$$

Plane strain $$\Delta W_{\text{closing}} = \frac{B(1-\nu^2)K_\text{I}(a)K_\text{I}(a+\Delta a)\Delta a}{E}$$

# $\mathcal{G}$ is univocally related to $K$

$$\mathcal{G} = \lim_{\Delta A \to 0} -\frac{\Delta \mathcal{P}}{\Delta A} = \lim_{\Delta A \to 0} -\frac{\Delta W_e}{\Delta A} = \lim_{\Delta (Ba) \to 0} \frac{\Delta W_{\text{closing}}}{\Delta (Ba)} = \frac{K_I^{\,2}(a)}{E^*}$$

Plane stress : $E^* = E$

Plane strain : $E^* = E/(1-\nu^2)$

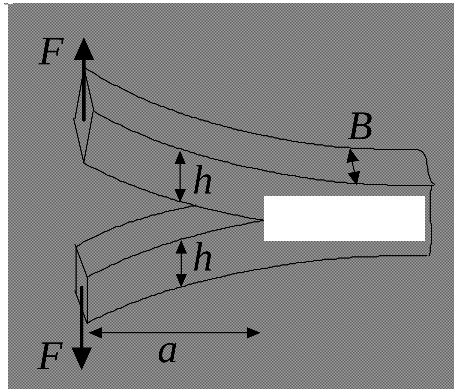
In general

$$\mathcal{G} = \frac{1}{E^*}\left( K_I^{\,2} + K_{II}^{\,2} + \frac{K_{III}^{\,2}}{1-\nu} \right)$$

# $\mathcal{G}$ is univocally related to *K*

*If $\mathcal{G}$ can be determined, hence we directly get an expression for K !!*

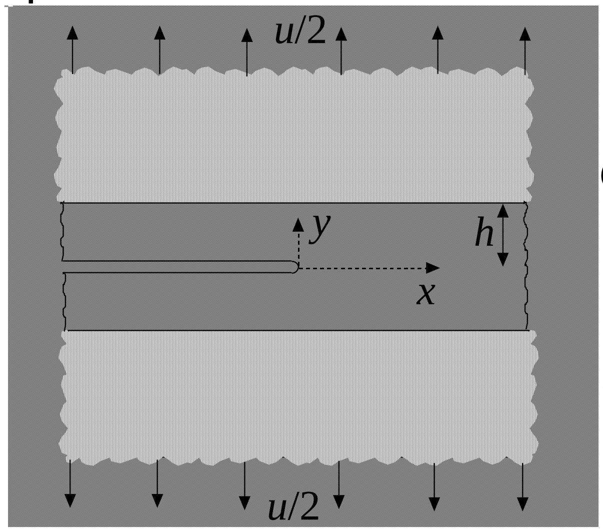
Example 1 : Double cantilever beam




$$\mathcal{G} = \frac{12F^2A^2}{EB^4h^3} = \frac{12F^2a^2}{EB^2h^3}$$

$$K = 2\frac{Fa}{Bh}\sqrt{\frac{3}{h}}$$

Example 2 : Semi-infinite crack in a strip held in rigid grips




$$\mathcal{G} = \frac{1}{4}\frac{E}{1-\nu^2}\frac{u^2}{h}$$

$$K = \frac{E}{2\sqrt{1-\nu^2}}\frac{u}{\sqrt{h}}$$

Unfortunately, only a limited number of configuration allow analytical determination of an expression for $\mathcal{G}$

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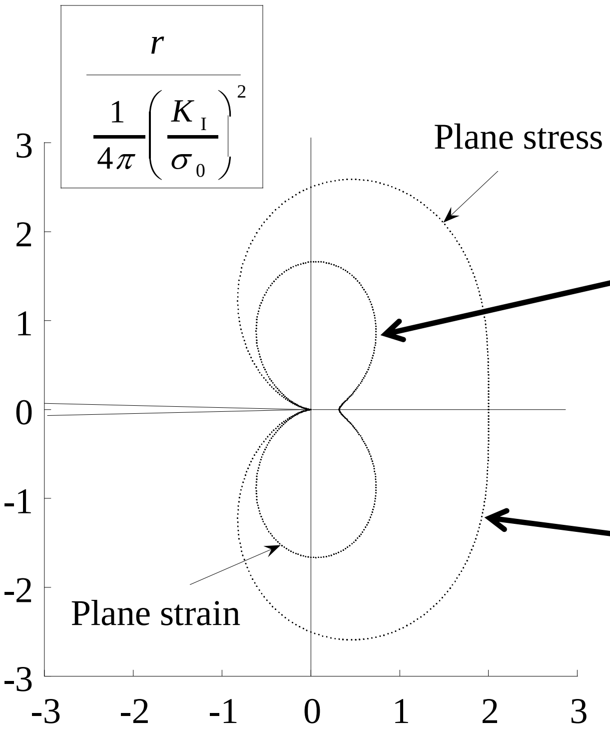
# The «non-linear» zone

If « non-linear » means “plastic yielding”

von Mises : $$\bar{\sigma} = \sqrt{\frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\right] + 3\tau_{xy}^2}$$




$$r_Y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_0}\right)^2\left((1-2\nu)^2(1+\cos\theta) + \frac{3}{2}\sin^2\theta\right)$$

$$r_Y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_0}\right)^2\left(1+\cos\theta + \frac{3}{2}\sin^2\theta\right)$$

# More exact solution from FE calculations

$$r_{\mathrm{Y}}^{\text{plane strain}} = \frac{1}{3\pi}\left(\frac{K_{\mathrm{I}}}{\sigma_0}\right)^2$$

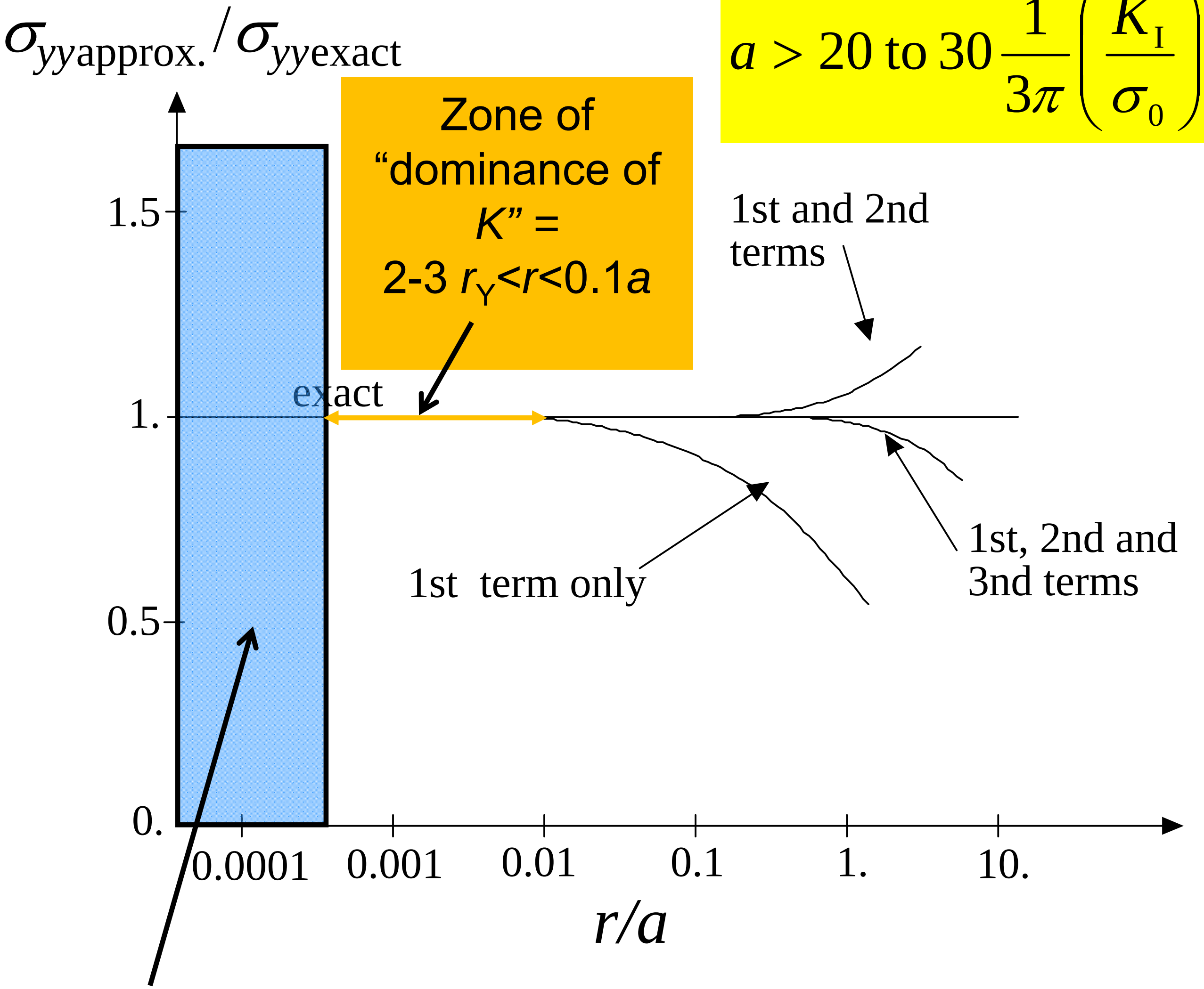
(plane strain)

$$r_{\mathrm{Y}}^{\text{plane stress}} = \frac{1}{\pi}\left(\frac{K_{\mathrm{I}}}{\sigma_0}\right)^2$$

(plane stress).

# Minimum specimen size for SSY

$$a > 20 \text{ to } 30 \frac{1}{3\pi}\left(\frac{K_I}{\sigma_0}\right)^2 = 2 \text{ to } 3\left(\frac{K_I}{\sigma_0}\right)^2$$




Zone of non-linear processes = 2-3 $r_Y$

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**Let's talk now about the resistive force (the material property)**

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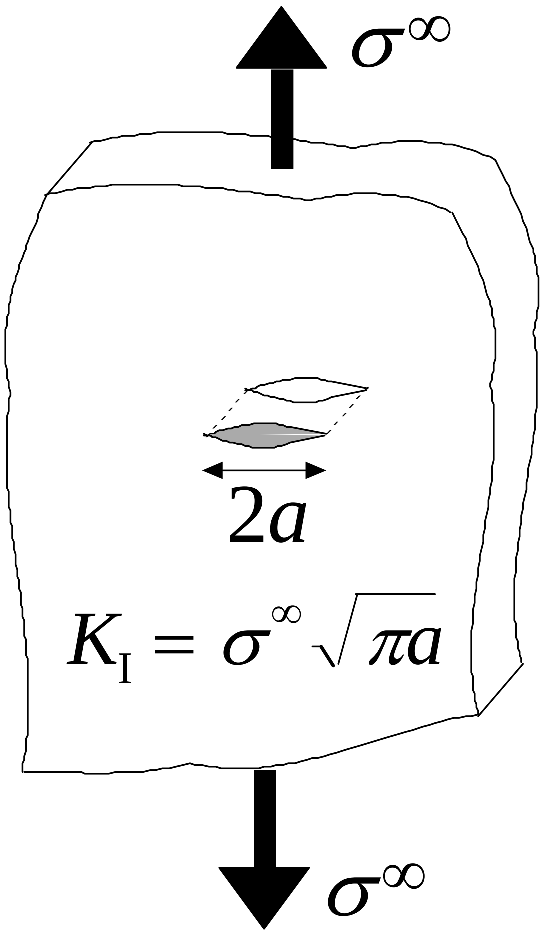
# The Griffith criterion




$$\mathcal{G} \geq 2\gamma_S$$

$$K_I \geq \sqrt{2\gamma_S E^*}$$

$$\sigma_c \geq \sqrt{\frac{2\gamma_S E}{\pi a\left(1-\nu^2\right)}}$$ (plane stain)

$$\sigma_c \geq \sqrt{\frac{\mathcal{G}_{IC} E}{\pi a\left(1-\nu^2\right)}}$$

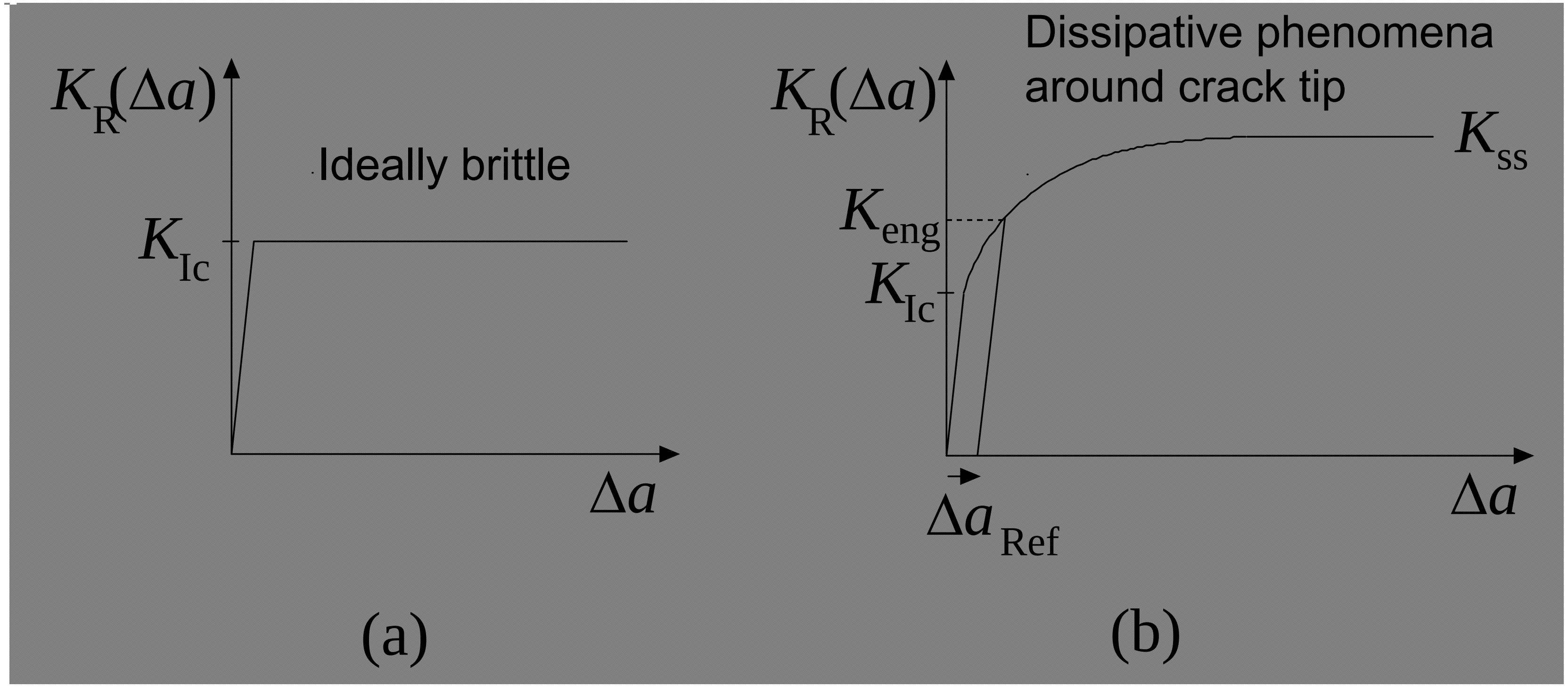
# Fracture toughness $\mathcal{G}_{Ic}$ or $K_{Ic}$
# = crack initiation resistance

Condition for initiation ofcracking :

$$\mathcal{G}_I \geq \mathcal{G}_{Ic} \text{ or } K_I \geq K_{Ic} \,.$$

For a given material, $K_{Ic}$ and $\mathcal{G}_{Ic}$ vary as a function of microstructure, temperature, velocity of crack propagation and environmental conditions – this where mechanics and materials science must discuss.

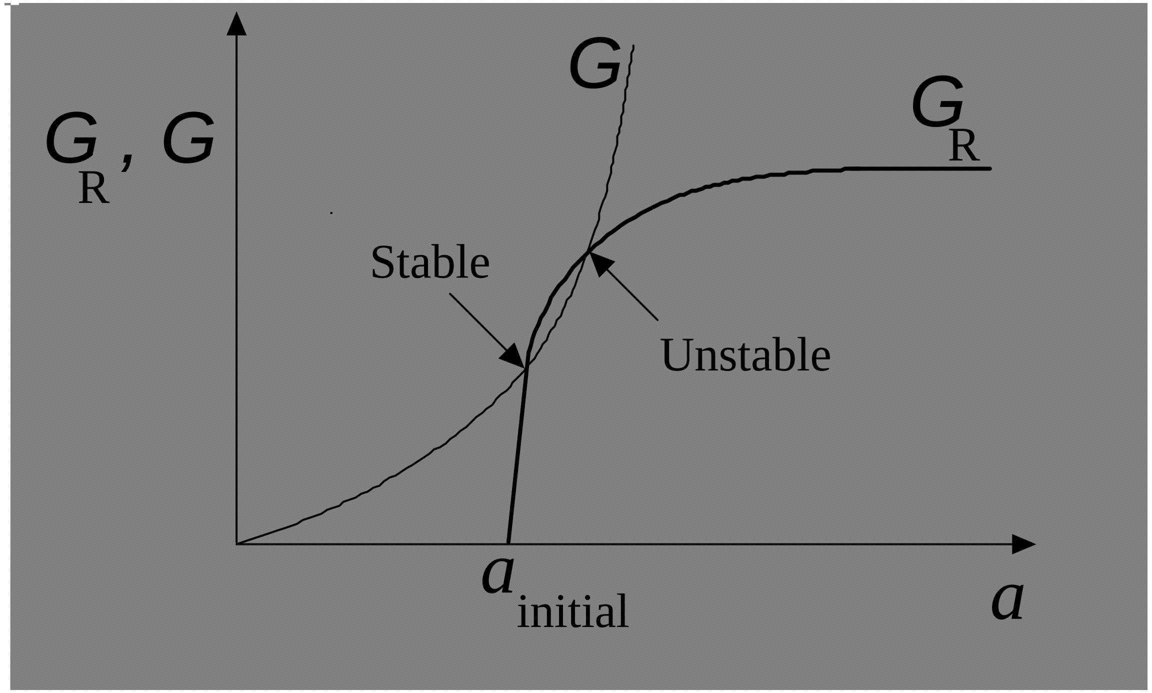
# *R*-curve : Crack extension resistance curve




Condition for cracking :

$\mathcal{G}_I \geq \mathcal{G}_{Ic}\,(\Delta a)$ or $K_I \geq K_{Ic}\,(\Delta a)$.

# General condition for stable cracking




2 conditions :

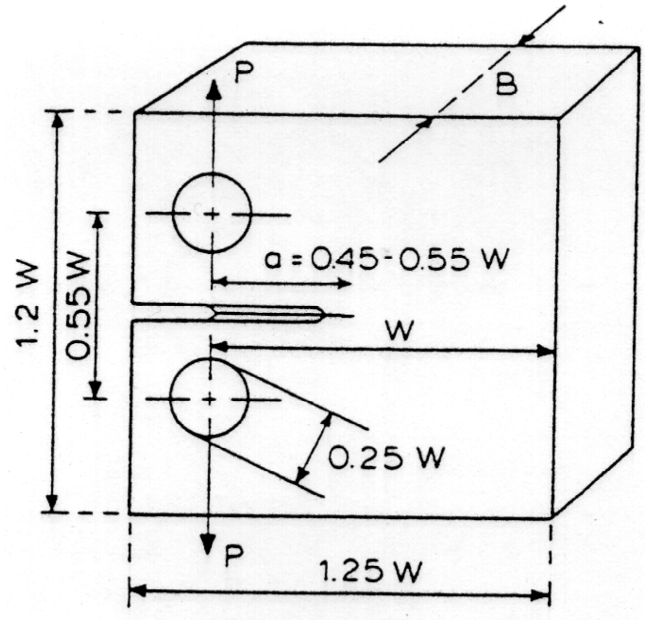
$\mathcal{G} = \mathcal{G}_R$

and

$$\frac{\partial \mathcal{G}}{\partial a} < \frac{\partial \mathcal{G}_R}{\partial a}$$

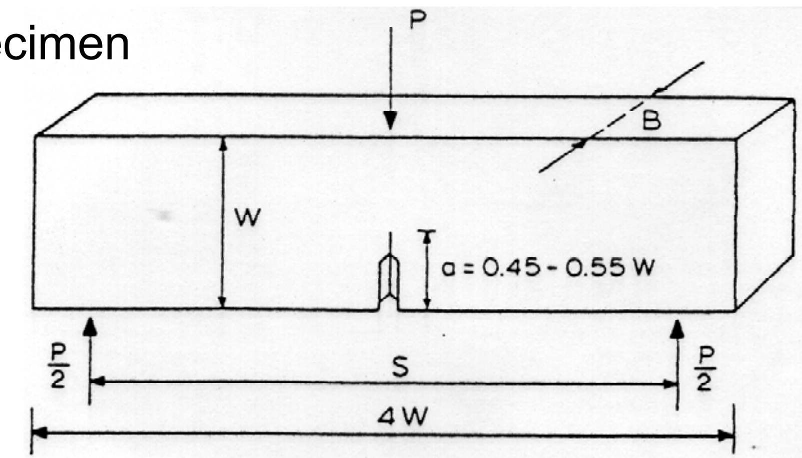
# Test methods for characterizing the fracture toughness of a material

"Compact tension" (CT) specimen




***Necessity to pre-crack the specimens; most often fatigue is needed***

Single edge notched beam (SENB) specimen



# Conditions for correct measurement of $K_{Ic}$

- Ideally sharp starting crack obtained by fatigue loading
- Small enough non-linear zone

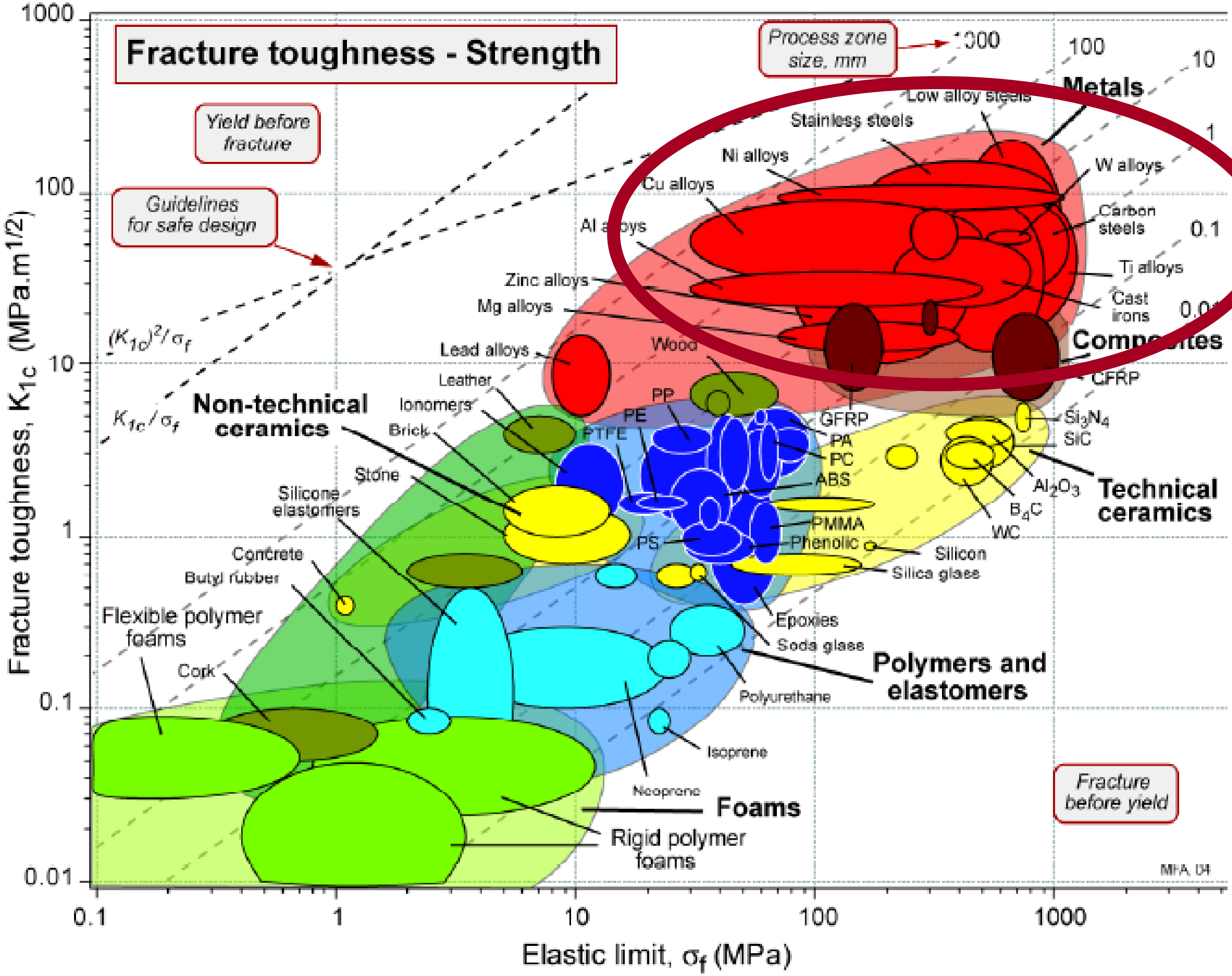
$$W - a > 2.5\left(\frac{K_{\mathrm{Ic}}}{\sigma_0}\right)^2$$

- Negligible contribution of plane stress

$$B > 2.5\left(\frac{K_{\mathrm{I}}}{\sigma_0}\right)^2$$

!! Large fracture toughness materials require very large specimens
e. g. low carbon steel : $(K_{Ic}/\sigma_0)^2 = 0.36$m !

# Fracture toughness remains one of the best assets of metals !



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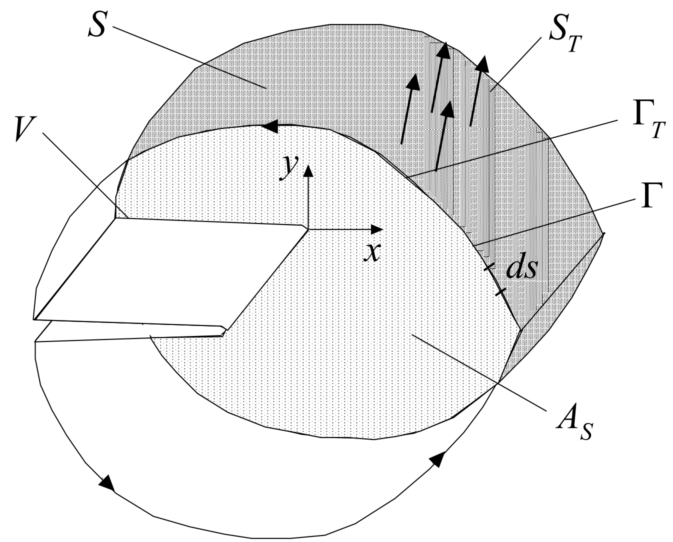
# J integral (Rice, 1968)

$$J = -\frac{\partial P}{\partial A} \quad \left(\mathrm{J/m^2}\right)$$

$$= \int \left( W_V n_x - n_i \sigma_{ij} \frac{\partial u_j}{\partial x} \right) ds$$

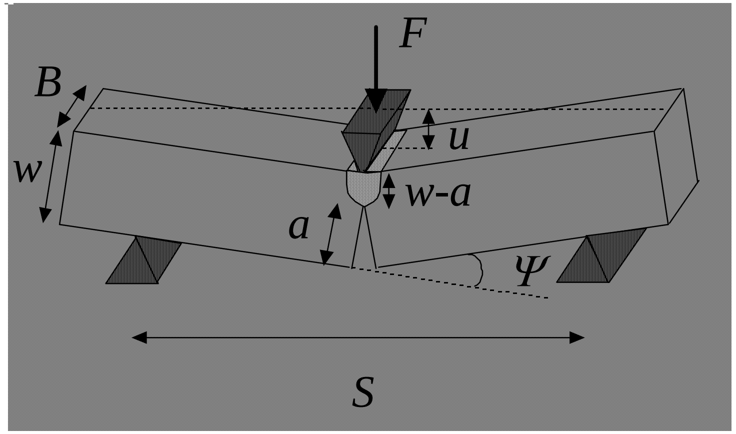



*valid for radial loadings*

$W_v$ is the strain energy density

$n_j$ = components of unit vector along outward normal to $\Gamma$

$$J = \frac{\eta}{B(w-a)} \int_0^F F du$$




$$J_{\mathrm{SENBdeep}} = \frac{2}{B(w-a)} \int_0^F F du$$

# HRR fields (Hutchinson, Rice and Rosengren, 1968)

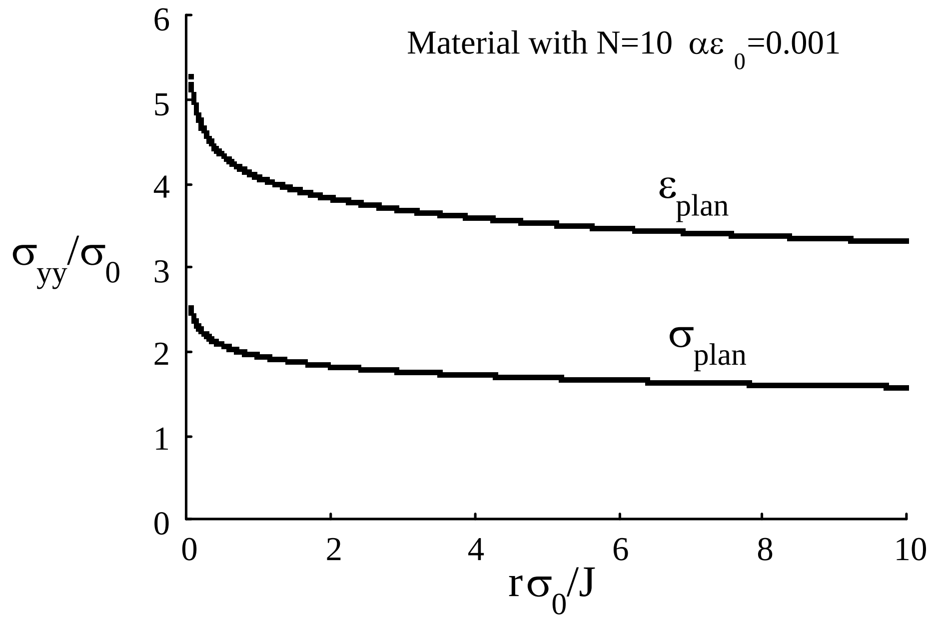
$$\frac{\varepsilon}{\varepsilon_0} = \alpha\left(\frac{\sigma}{\sigma_0}\right)^N$$

J2 deformation theory
(non linear elastic response)

$$\sigma_{ij} = \sigma_0\left(\frac{J}{\alpha\sigma_0\varepsilon_0 I_N r}\right)^{\frac{1}{N+1}}\tilde{\sigma}_{ij}(\theta, N)$$

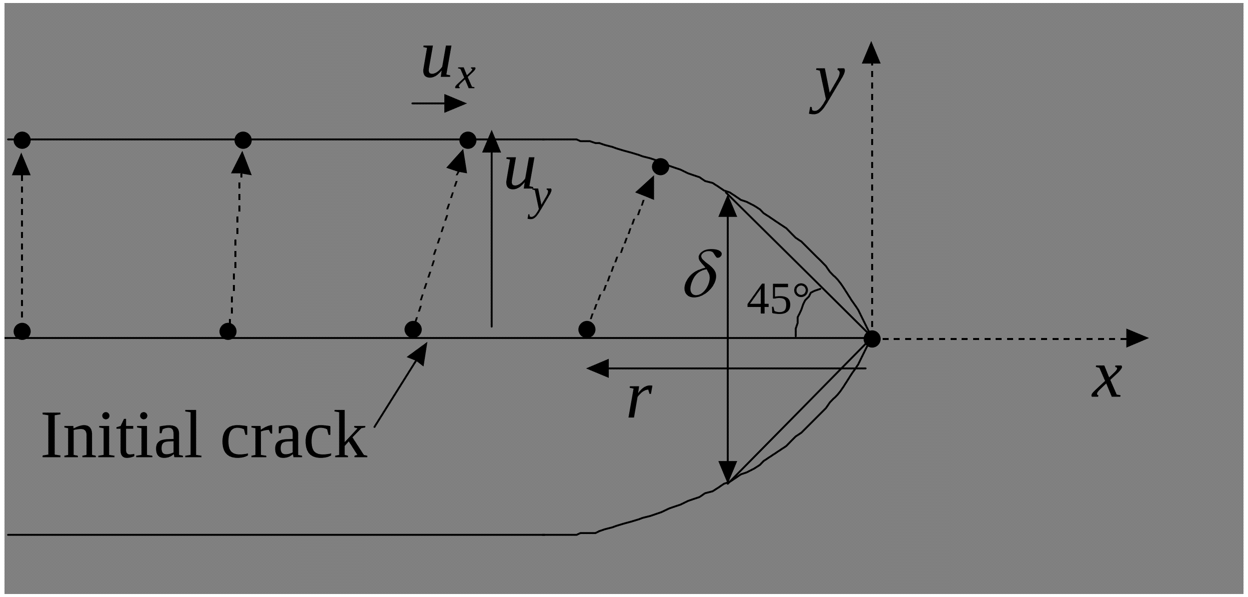
$$\overline{\sigma} = \sigma_0\left(\frac{J}{\alpha\sigma_0\varepsilon_0 I_N r}\right)^{\frac{1}{N+1}}\tilde{\overline{\sigma}}(\theta, N)$$

$$\varepsilon_{ij} = \alpha\varepsilon_0\left(\frac{J}{\alpha\sigma_0\varepsilon_0 I_N r}\right)^{\frac{N}{N+1}}\tilde{\varepsilon}_{ij}(\theta, N)$$




*See HHR tables by Fong Shih, 1983 (Brown University)*

# Crack tip opening displacement

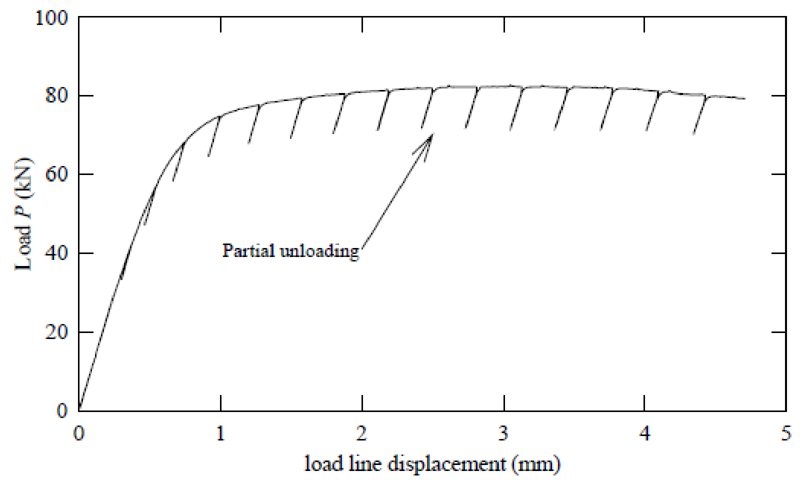


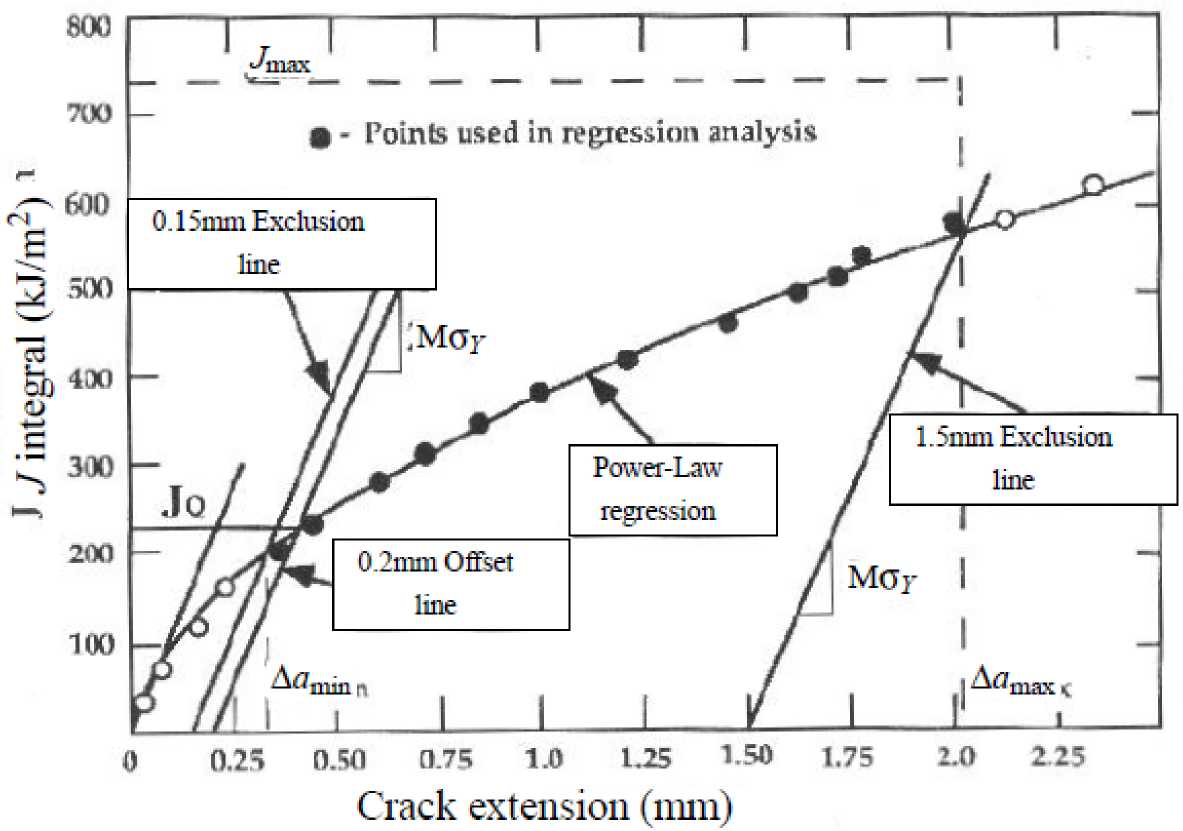

$$u_i = \alpha \varepsilon_0 r \left( \frac{J}{\alpha \sigma_0 \varepsilon_0 I_N r} \right)^{\frac{N}{N+1}} \tilde{u}_i(\theta, N)$$

$$\delta = d(\alpha \varepsilon_0, N) \frac{J}{\sigma_0}$$

Fracture toughness can also be defined as critical CTOD $\delta_c$

# JR curve
## *main goal is to determine $J_{Ic}$*



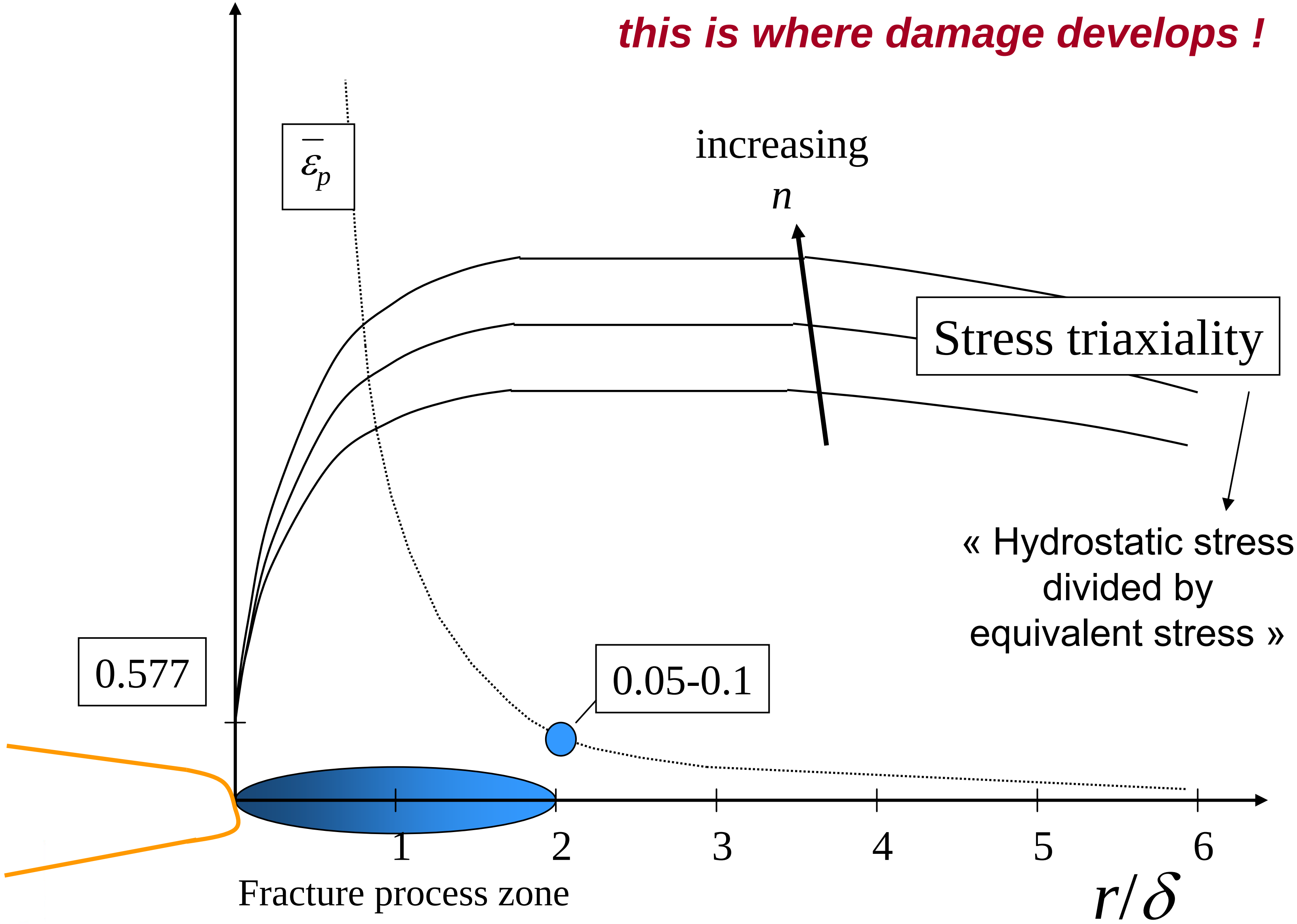




## Standardized method

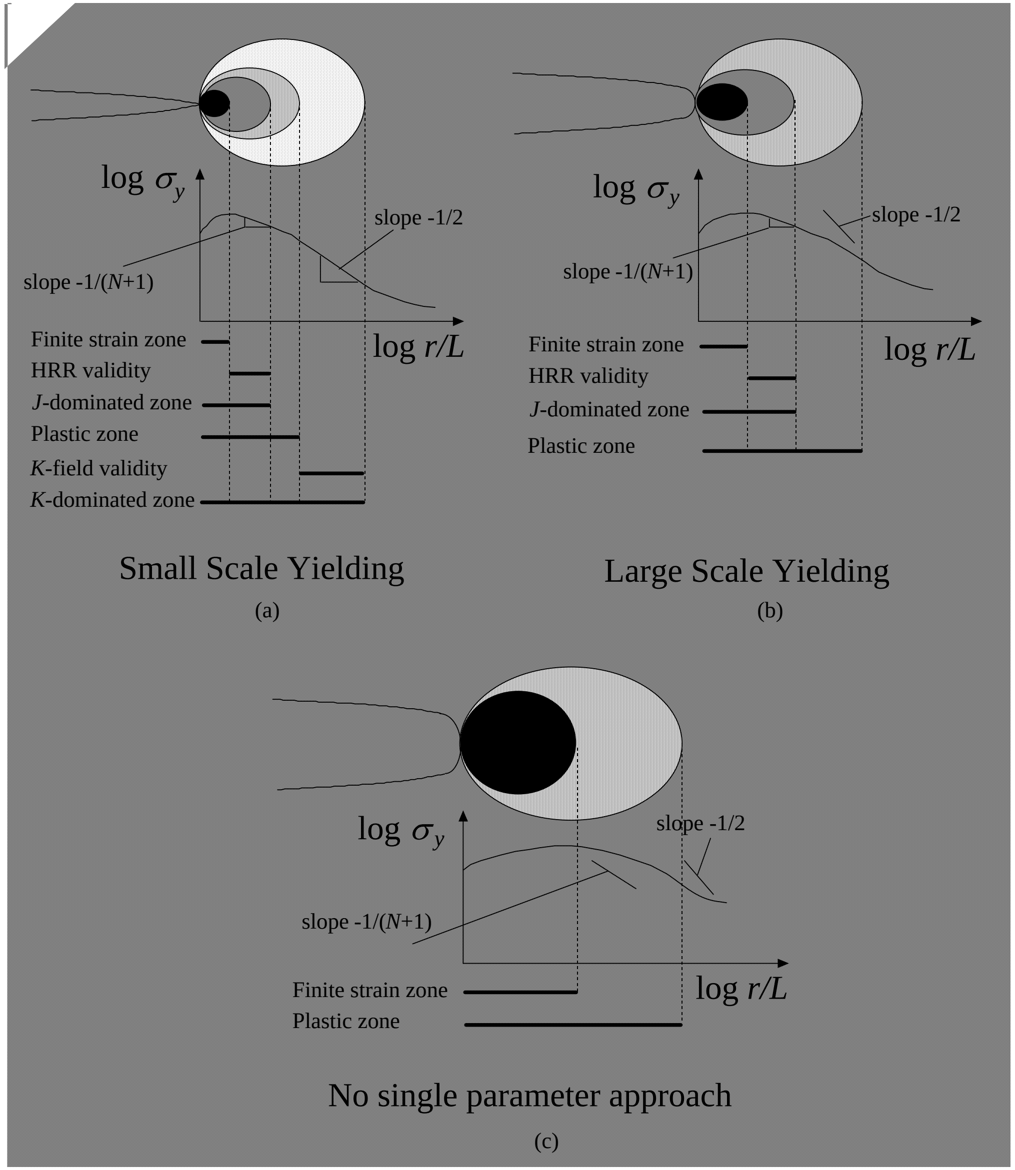
From B. Tanguy, CEA

# The HRR fields and J approach do not account for the presence of a finite strain zone

*this is where damage develops !*



# Small scale yielding, large scale yielding, or general yielding ahead of crack tip



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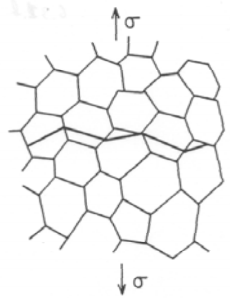
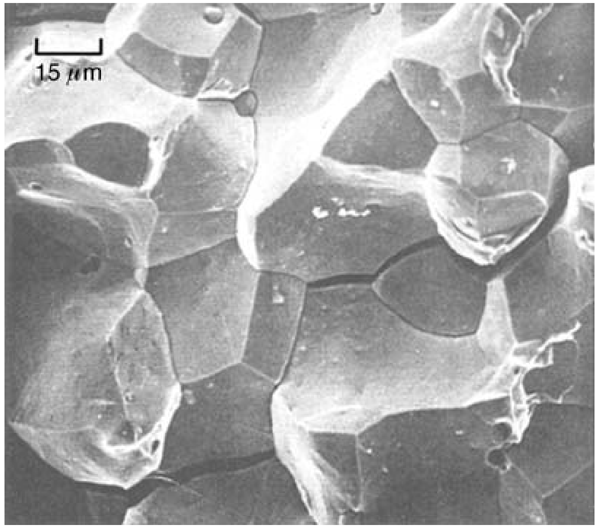
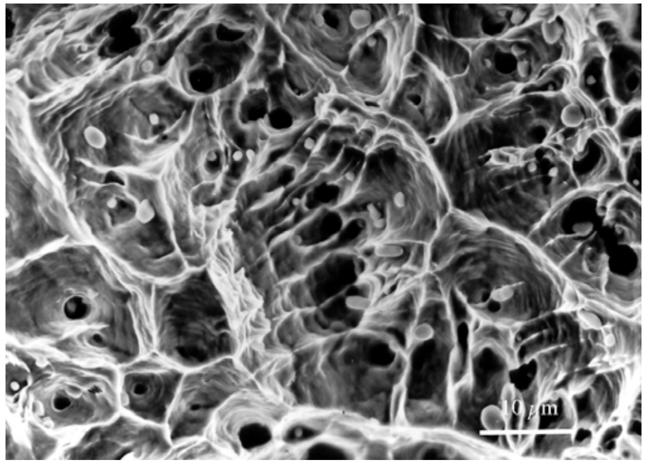
# Two main mechanisms of fracture in metals

- "Brittle fracture" proceeds essentially by cleavage through grains along crystalline directions



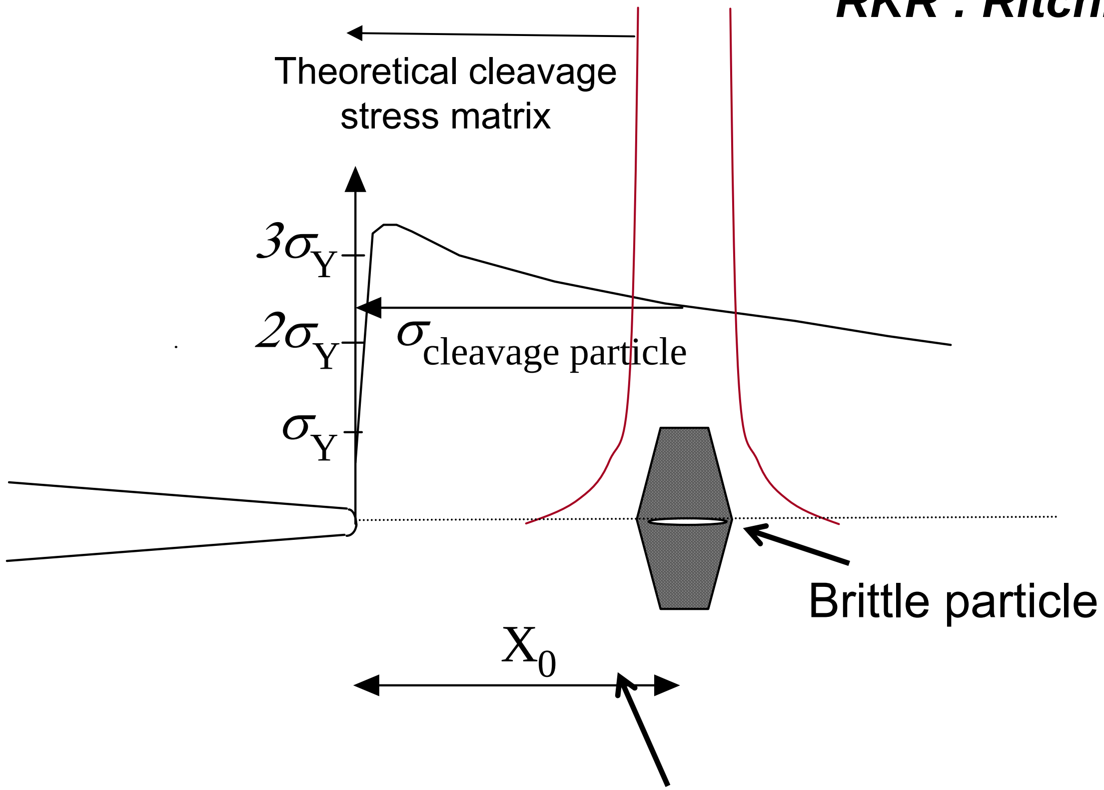




- "Ductile fracture" proceeds essentially by plastic growth of voids nucleating ahead of the crack.



# RKR model for the cleavage fracture of metals

***RKR : Ritchie, Knott and Rice***




Average distance between brittle particles

*The speed of the microcrack when reaching the interface with the matrix is large enough for preventing dislocations to operate within the matrix in materials such as BCC crystals which exhibit a significant sensitivity .*

⟶ "crack injection" backward toward the crack tip

# RKR model

$$\sigma_y = \sigma_0 \left( \frac{J}{\alpha \varepsilon_0 \sigma_0 I_N r} \right)^{\frac{1}{N+1}} \tilde{\sigma}_y(0°, N)$$

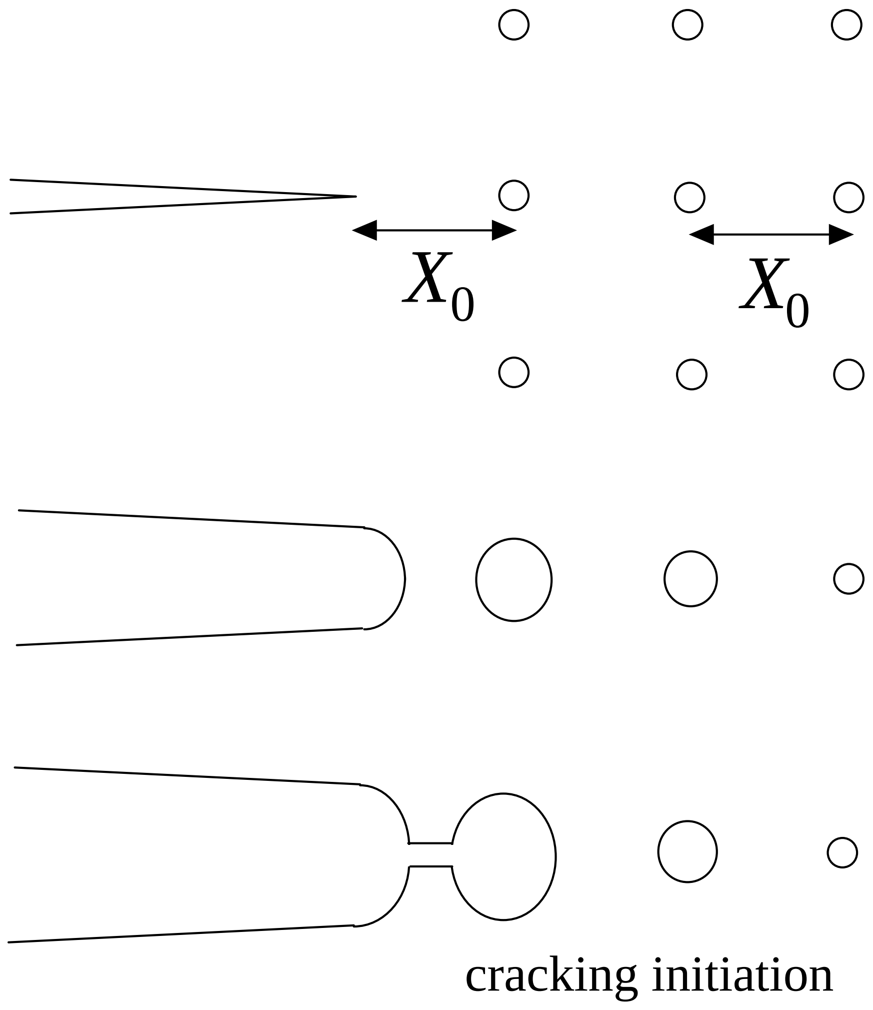
$= \sigma_\mathrm{c}$ of a brittle particle at distance $r = X_0$

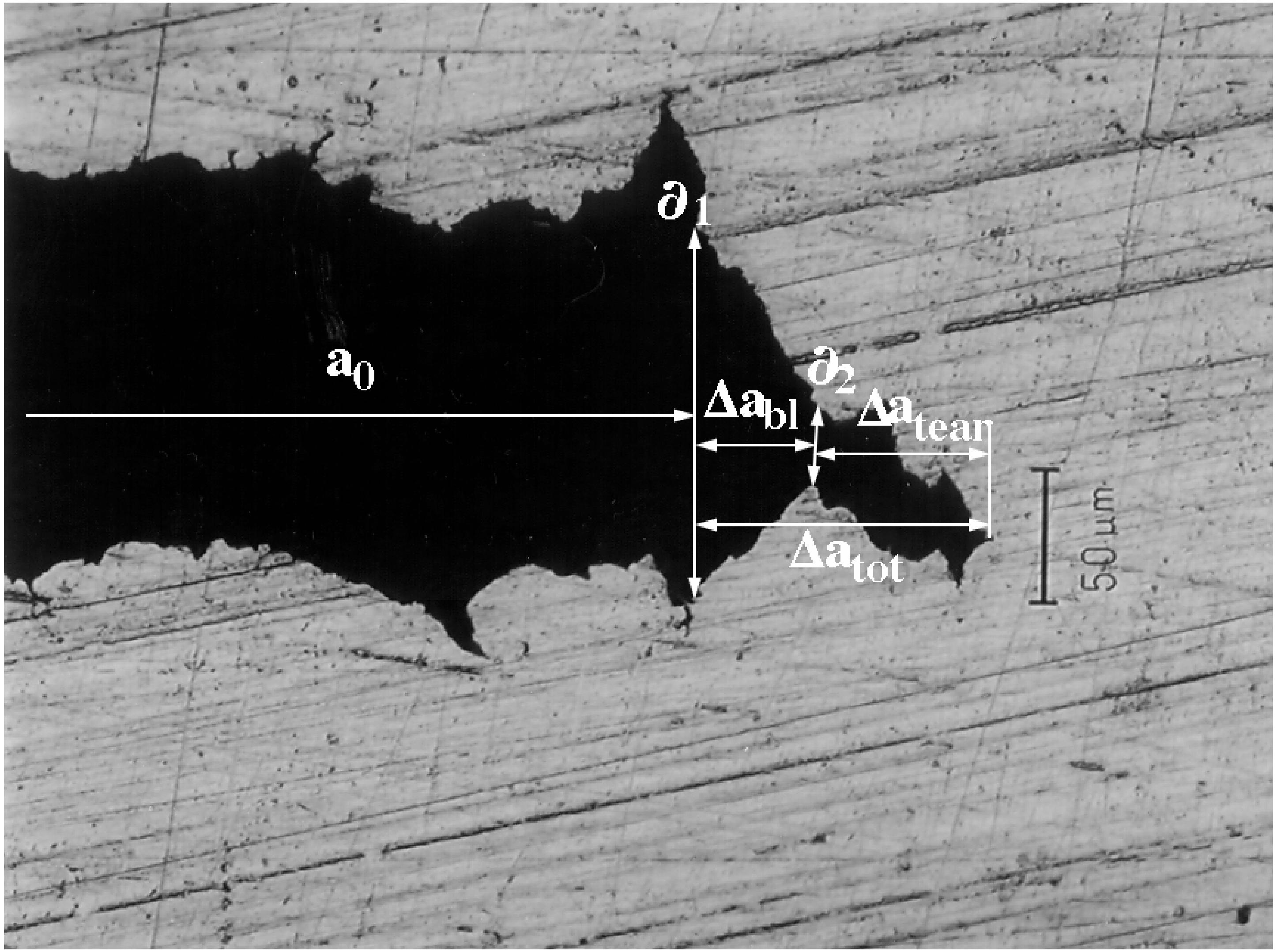
$$J_\mathrm{Ic} = \frac{\sigma_c^{N+1}}{\sigma_0^N} X_0 \frac{\sigma_0}{E} F(N, \alpha) \propto \frac{\sigma_c^{N+1}}{\sigma_0^{N-1}} X_0$$

$\Rightarrow$ in case of brittle mechanism dominated by initiation (we neglect arrest by grain boundaries, etc), toughness increases when

- $\sigma_\mathrm{c}$ increases
- $\sigma_0$ decreases (at constant grain size)
- $X_0$ (or particle volume fraction) decreases

# Mechanism of ductile fracture in metals




∂1
a0
∂2
Δabl
Δatear
Δatot
50 µm

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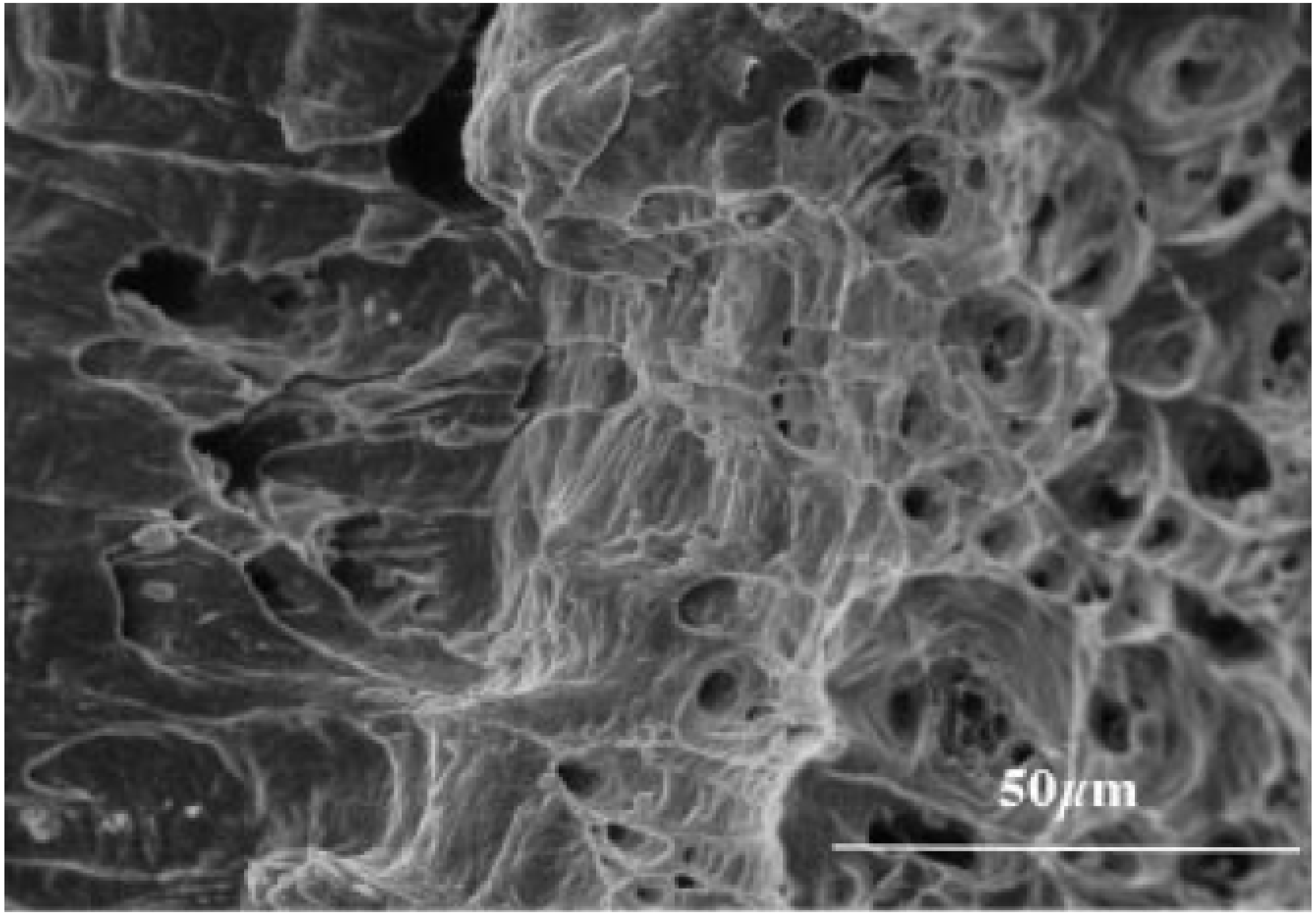
Fatigue zone

Blunting zone = stretch zone width (≈ $\delta_c/2$)

Ductile tearing



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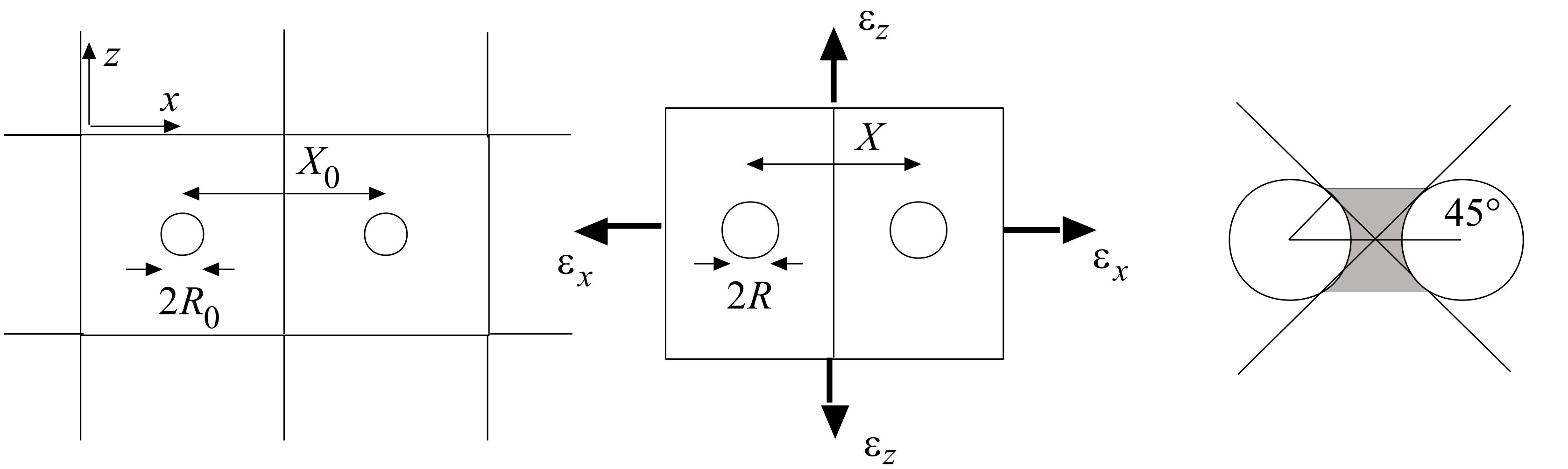
# Very simple model for growth and coalescence of voids in metals




a. initial geometry

b. current geometry

c. onset of coalescence

$$X = X_0 \exp(\varepsilon_x)$$

$$\frac{\mathrm{d}R}{R} = 0.43 \exp\left(\frac{3}{2}\frac{\sigma_\mathrm{h}}{\overline{\sigma}}\right) \mathrm{d}\overline{\varepsilon}^\mathrm{p}$$

*Rice and Tracey void growth model (1969)*

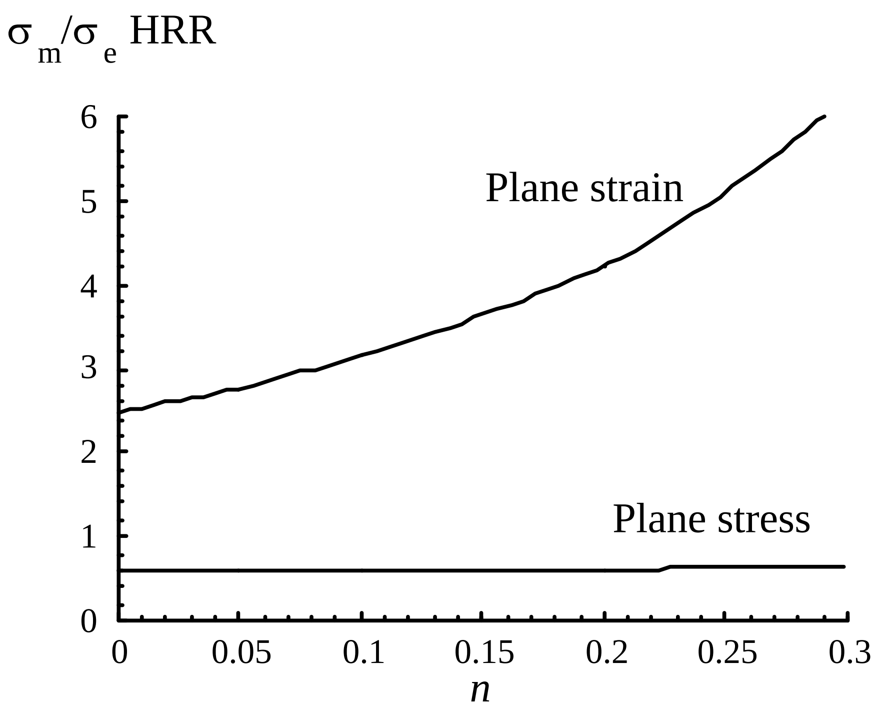
$$X = \lambda R = 2\sqrt{2}R$$

*Brown and Embury coalescence criterion (1973)*

# Stress triaxiality ahead of crack tip (HRR)

$$T_{\mathrm{HRR}} = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3\bar{\sigma}} = \frac{\tilde{\sigma}_{11} + \tilde{\sigma}_{22} + \tilde{\sigma}_{33}}{3\tilde{\bar{\sigma}}}$$

Independent of distance *r*, but depends on hardening exponent *n*



## **Combine the three models for** providing an equation for calculating the strain at coalescence as a function of the initial void volume fraction $f_0$ and stress state

- void nucleation criterion to predict the strain at void nucleation $\varepsilon_c$

- Void growth model

$$\frac{\exp(\varepsilon_x)}{\lambda \exp\left[0.43\int_{\varepsilon_c}^{\varepsilon} \exp\left(\frac{3}{2}\frac{\sigma_h}{\overline{\sigma}}\right) d\overline{\varepsilon}^p\right]} = \frac{R_0}{X_0} = \left(\frac{3}{4\pi} f_0\right)^{1/3}$$

- Introduce current radius of the void in Brown-Embury condition

# Prediction of $J_{Ic}$ for ductile fracture

The strain at coalescence can thus be expressed

$$\frac{\exp\left(\varepsilon_x^{\mathrm{HRR}}\right)}{2\sqrt{2}\exp\left(0.43\left(\bar{\varepsilon}^{\mathrm{HRR}}-\varepsilon_{0nucl}\right)\exp\left(\frac{3}{2}T_{\mathrm{HRR}}\right)\right)}=\left(\frac{3}{4\pi}f_0\right)^{1/3}$$

, which yields

$$\varepsilon_x^{\mathrm{HRR}} - 0.43\bar{\varepsilon}^{\mathrm{HRR}}\exp\left(\frac{3}{2}T_{\mathrm{HRR}}\right)\approx 0.56+\frac{1}{3}\ln\left(f_0\right)$$

On average, the first void in the fracture process zone is located at a distance $X_0$ from the crack tip. This means that the criterion must be satisfied at $r = X_0$ :

$$\alpha\varepsilon_0\left(\frac{J}{\alpha\varepsilon_0\sigma_0 I_{\mathrm{N}} X_0}\right)^{N/N+1}\left(\tilde{\varepsilon}_x - 0.43\frac{2}{\sqrt{3}}\left|\tilde{\varepsilon}_x\right|0.\exp\left(\frac{3}{2}T_{\mathrm{HRR}}\right)\right)\approx 0.56+\frac{1}{3}\ln\left(f_0\right)$$

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# Prediction of $J_{Ic}$ for ductile fracture

$$J = J_{\mathrm{Ic}} \approx \alpha \varepsilon_0 \sigma_0 I_{\mathrm{N}} X_0 \left( \frac{0.56 + \frac{1}{3}\ln(\mathrm{f}_0)}{\alpha\varepsilon_0 \left( \tilde{\varepsilon}_x - 0.43 \frac{2}{\sqrt{3}} \left|\tilde{\varepsilon}_x\right| \exp\left(\frac{3}{2} T_{\mathrm{HRR}}\right)\right)} \right)^{\frac{N+1}{N}}$$

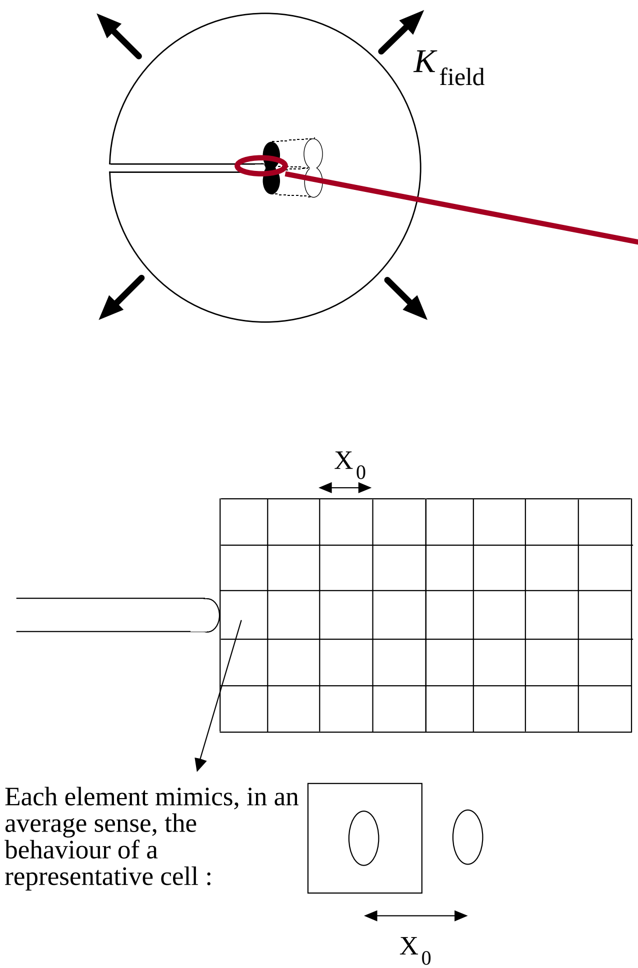
$$J_{\mathrm{Ic}} \approx \sigma_0 X_0 F\left(N, \frac{\sigma_0}{E}, f_0\right)$$

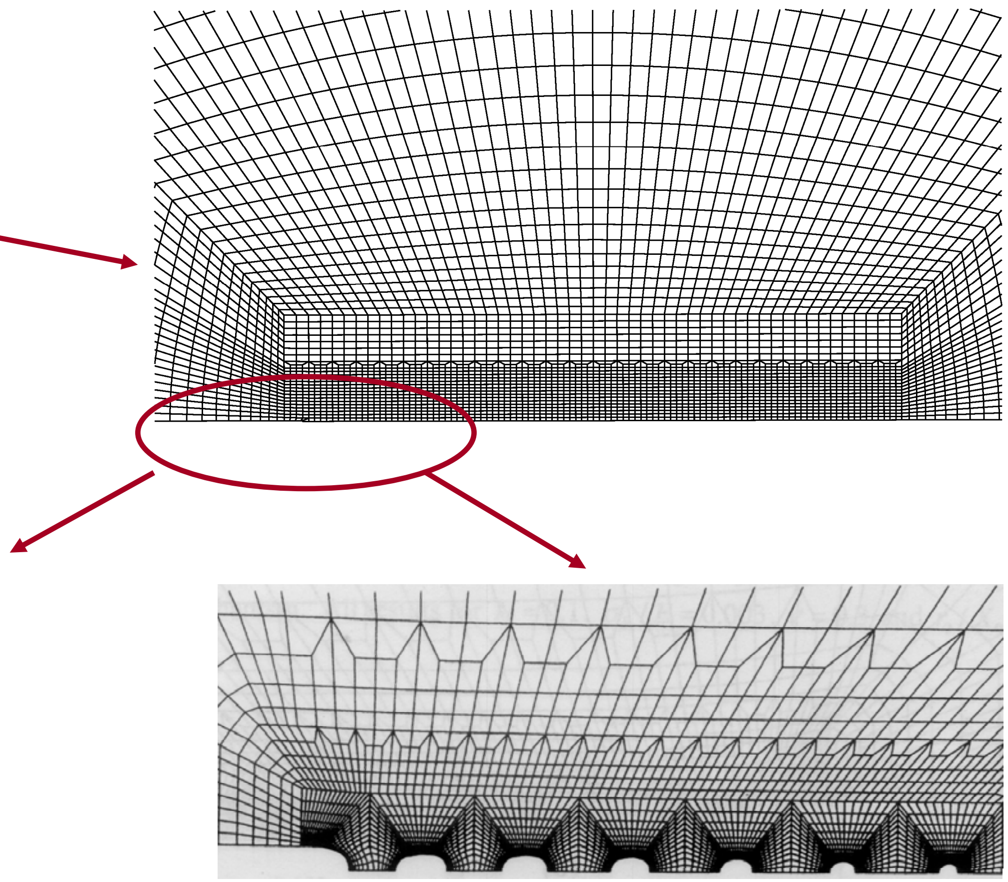
**Typically 0.3 < *F* < 5 … this is where the science and connection with the microstructure and hardening mechanisms is !**

*See e.g. Pardoen and Hutchinson, Acta Mater 2003*

**Very large energy dissipation : $J_{Ic}$ ranges from $10^4$ to $2.10^5$ $Jm^{-2}$, i.e. up to $10^5$ x $2\gamma_S$**

# Computational models for quantitative predictions



(from V. Tvergaard & J.W. Hutchinson, IJSS, 2002)

Pardoen, T., Hutchinson, J.W., 2003. Acta Mater. 51, 133-148.

PERGAMON

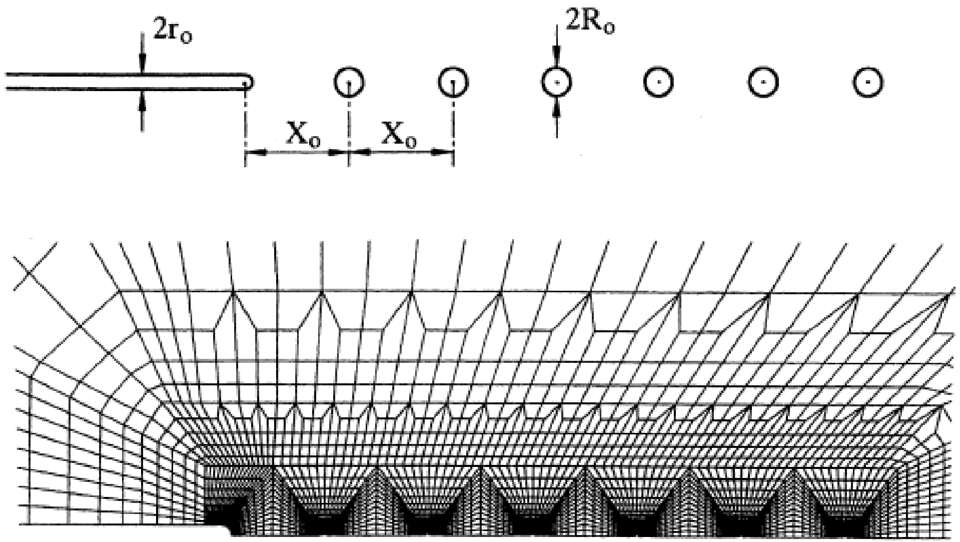
International Journal of Solids and Structures 39 (2002) 3581–3597

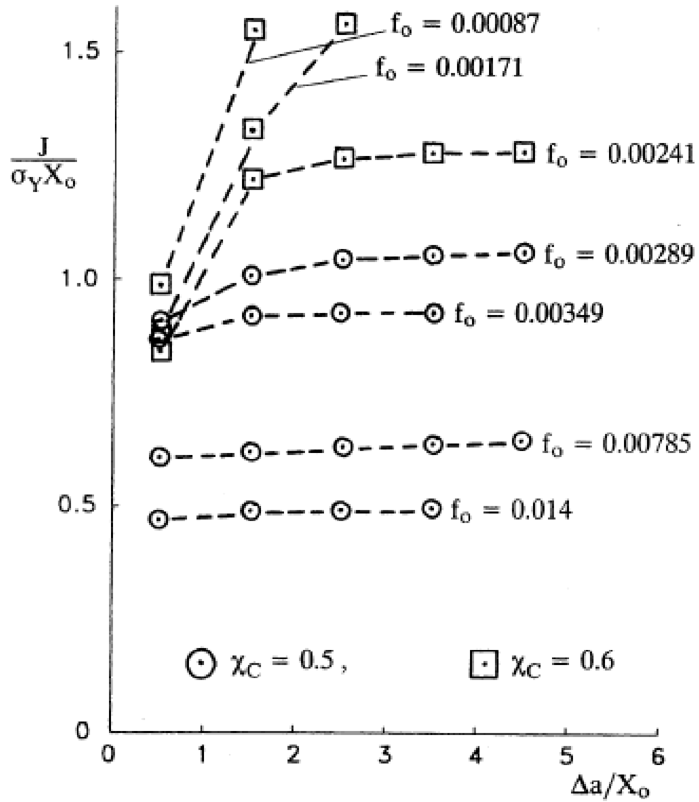
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

# Two mechanisms of ductile fracture: void by void growth versus multiple void interaction

Viggo Tvergaard [a], John W. Hutchinson [b,*]



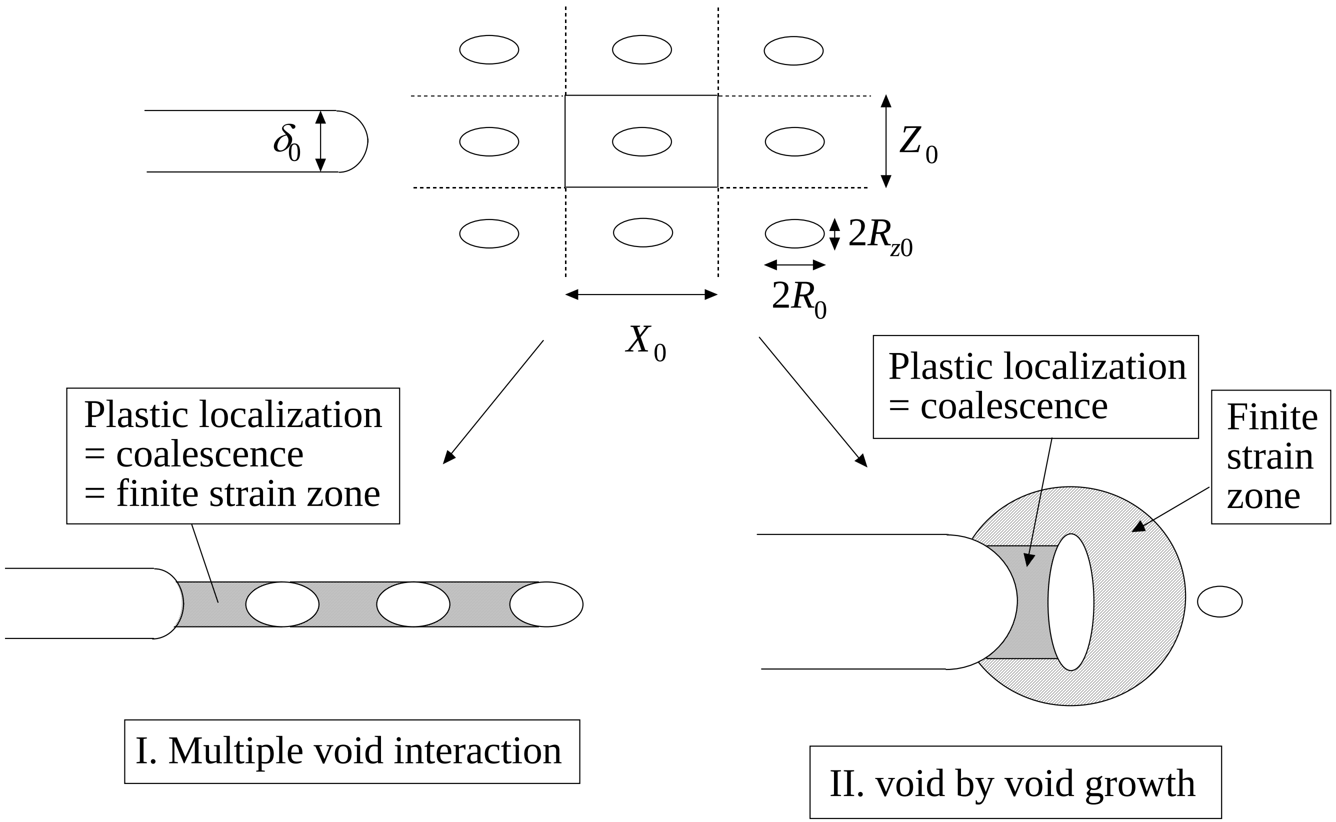



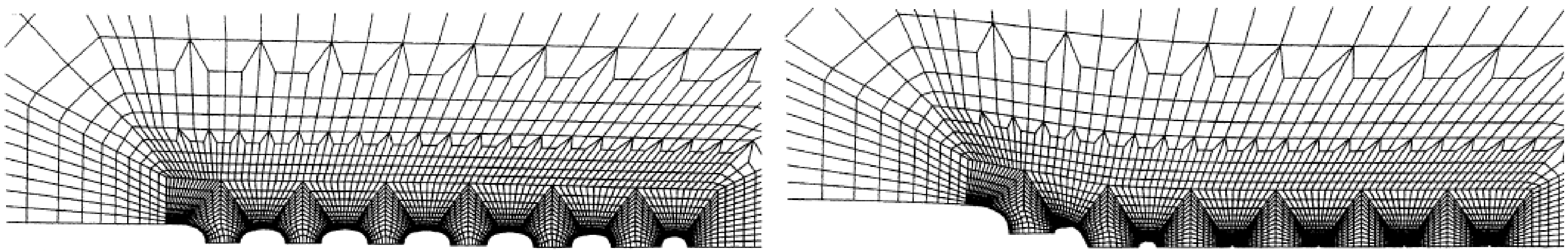

## See earlier works by McMeeking

Hom, C.L., McMeeking, R.M., 1989. Three-dimensional void growth before a blunting crack tip. J. Mech. Phys. Solids 37, 395–415.

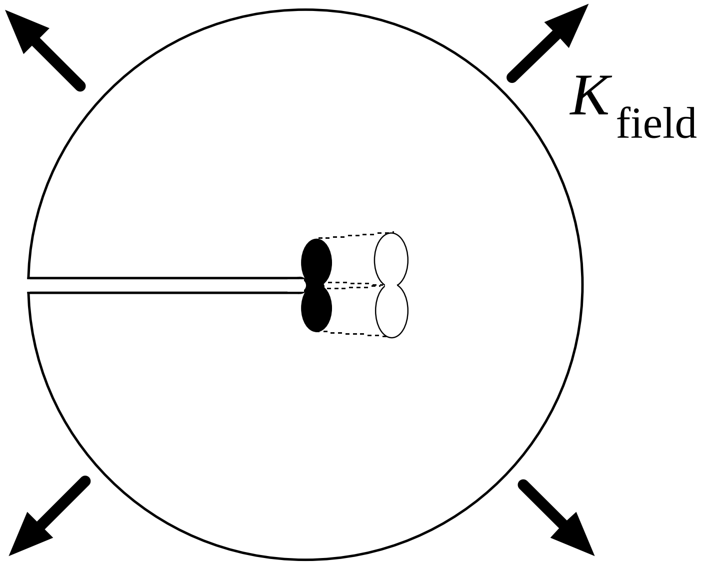
McMeeking, R.M., 1977. Finite deformation analysis of crack-tip opening in elastic–plastic materials and implications for fracture. J. Mech. Phys. Solids 25, 357–381.

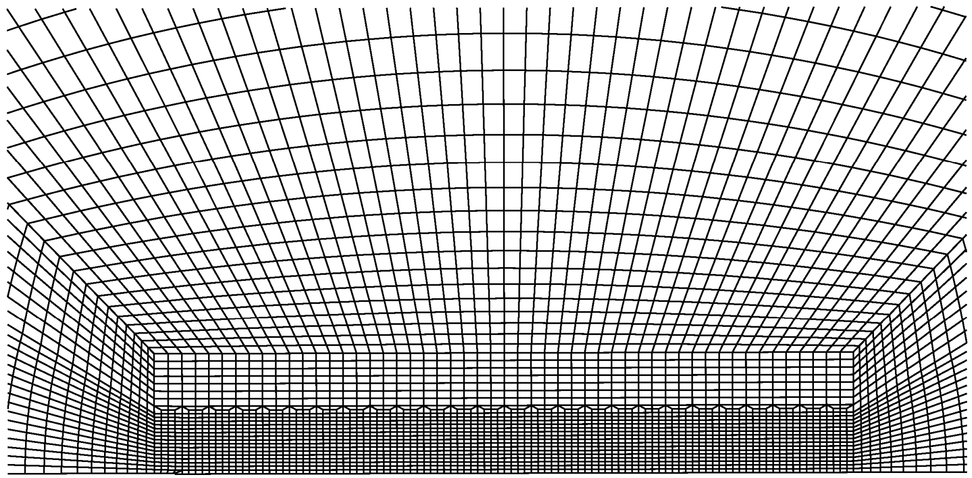
# Two “slightly different” ductile tearing mechanisms



# Small scale yielding analysis (infinite medium) of ductile fracture with advanced "Gurson" model



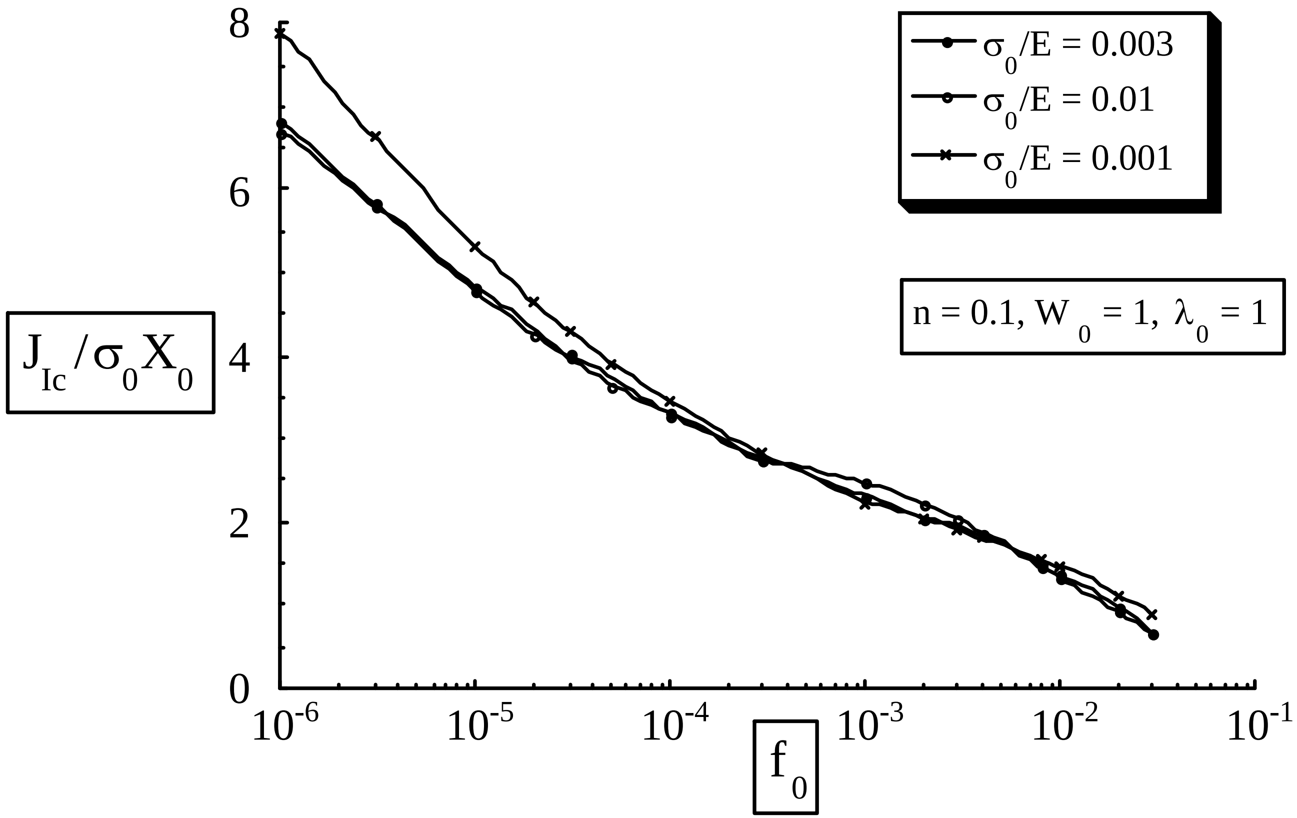
Model implemented in "ABAQUS Standard" through a User defined MATerial (UMAT), finite strain setting, fully implicit – or home code (Ph. D. Florence Scheyvaerts)

Pardoen, T., Hutchinson, J.W., 2003. Micromechanics-based model for trends in toughness of ductile metals. Acta Mater. 51, 133-148.

# Plane strain fracture toughness of ductile metallic alloys

## Negligible effect of $\sigma_0/E$ on $J_{Ic}/\sigma_0 X_0$

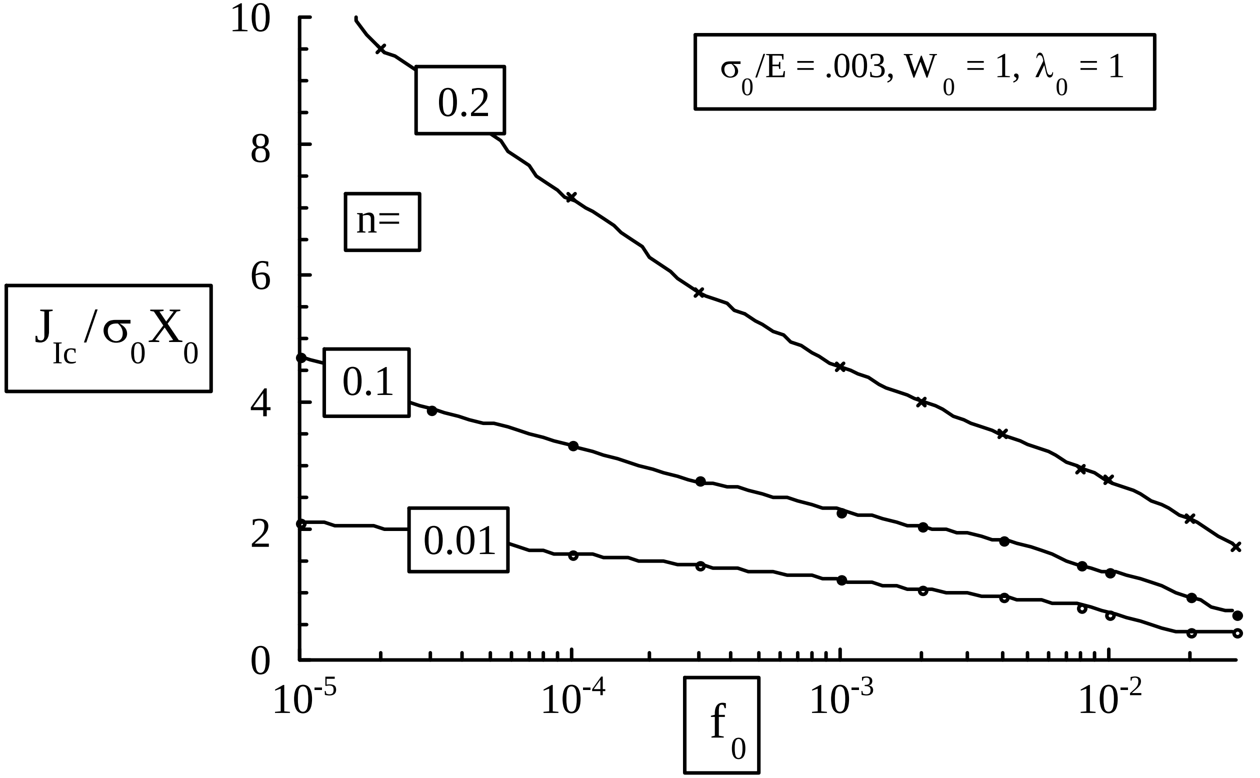
$$J_{Ic} = \sigma_0 X_0 F\left(n, \frac{\sigma_0}{E}, f_0, W_0, \lambda_0, \ldots\right)$$



# Plane strain fracture toughness of ductile metallic alloys

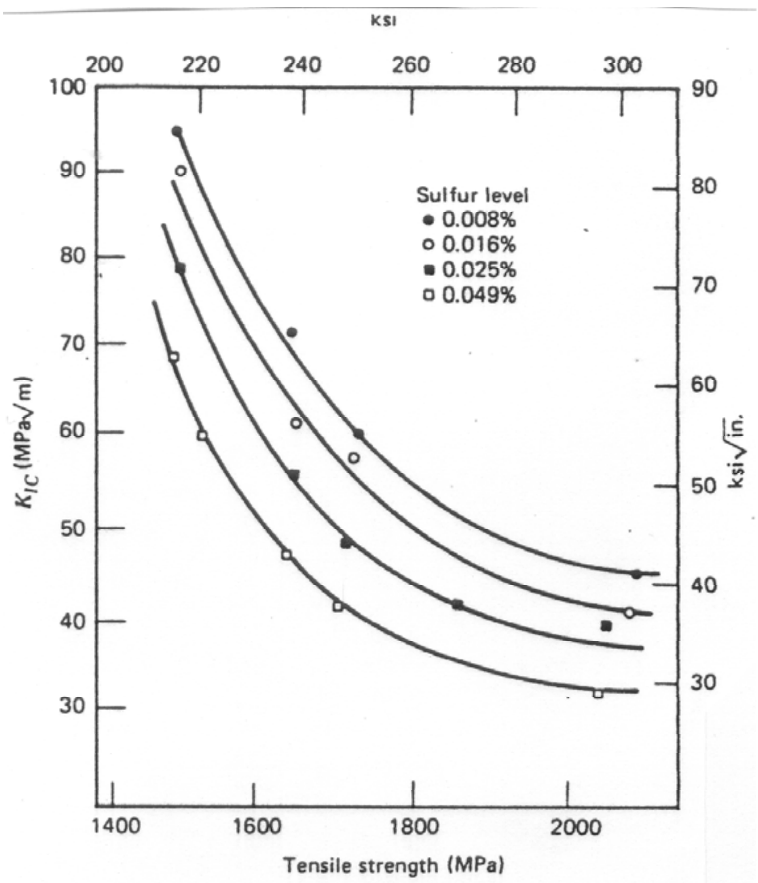
**Very strong effect of the strain hardening exponent *n* on $J_{Ic}/\sigma_0 X$**

$$J_{Ic} = \sigma_0 X_0 F\left(n, \frac{\sigma_0}{E}, f_0, W_0, \lambda_0, \ \ldots\right)$$

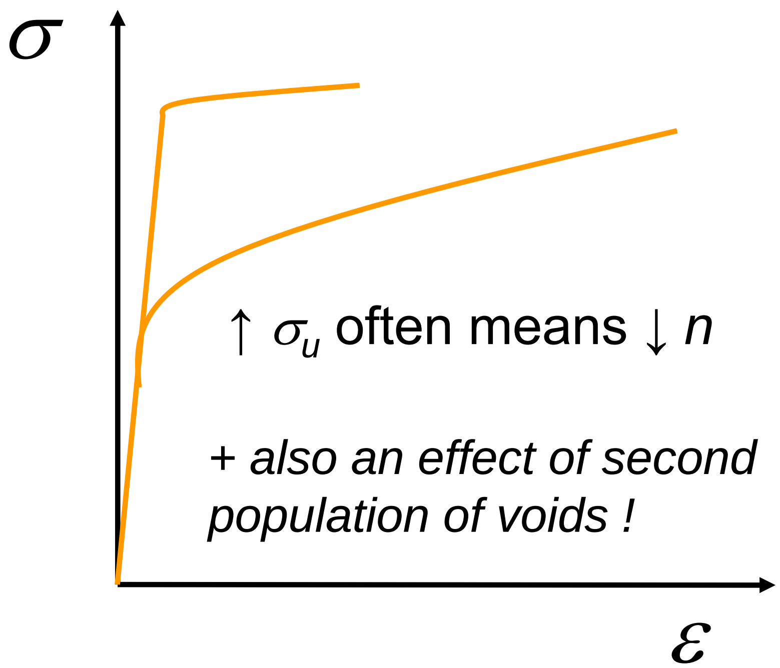


# Plane strain fracture toughness of ductile metallic alloys

**Large effect of strain hardening exponent partly explains why fracture toughness usually decreases with increasing strength – not a direct effect of the strength**



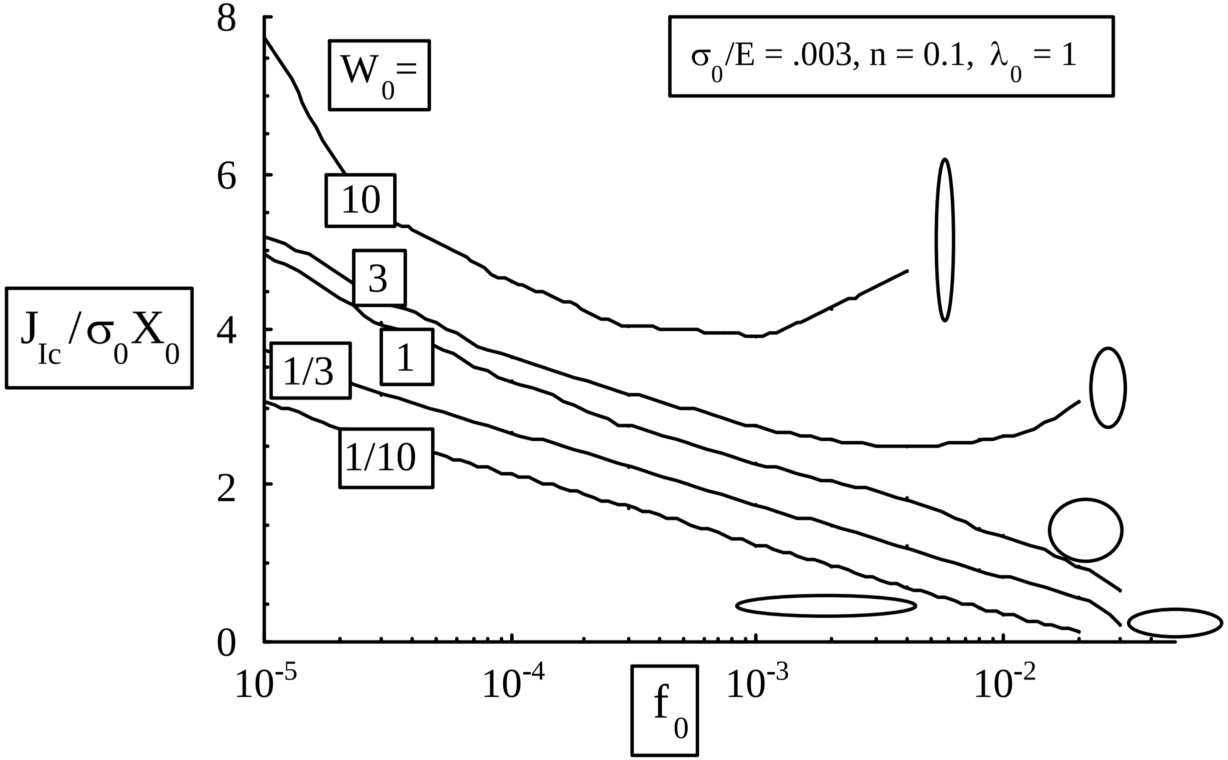

$$J_{Ic} = \sigma_0 X_0 F\left(n, \frac{\sigma_0}{E}, f_0, W_0, \lambda_0, \ldots\right)$$



# Plane strain fracture toughness of ductile metallic alloys

## Very strong effect of the initial void shape on $J_{Ic}/\sigma_0 X_0$

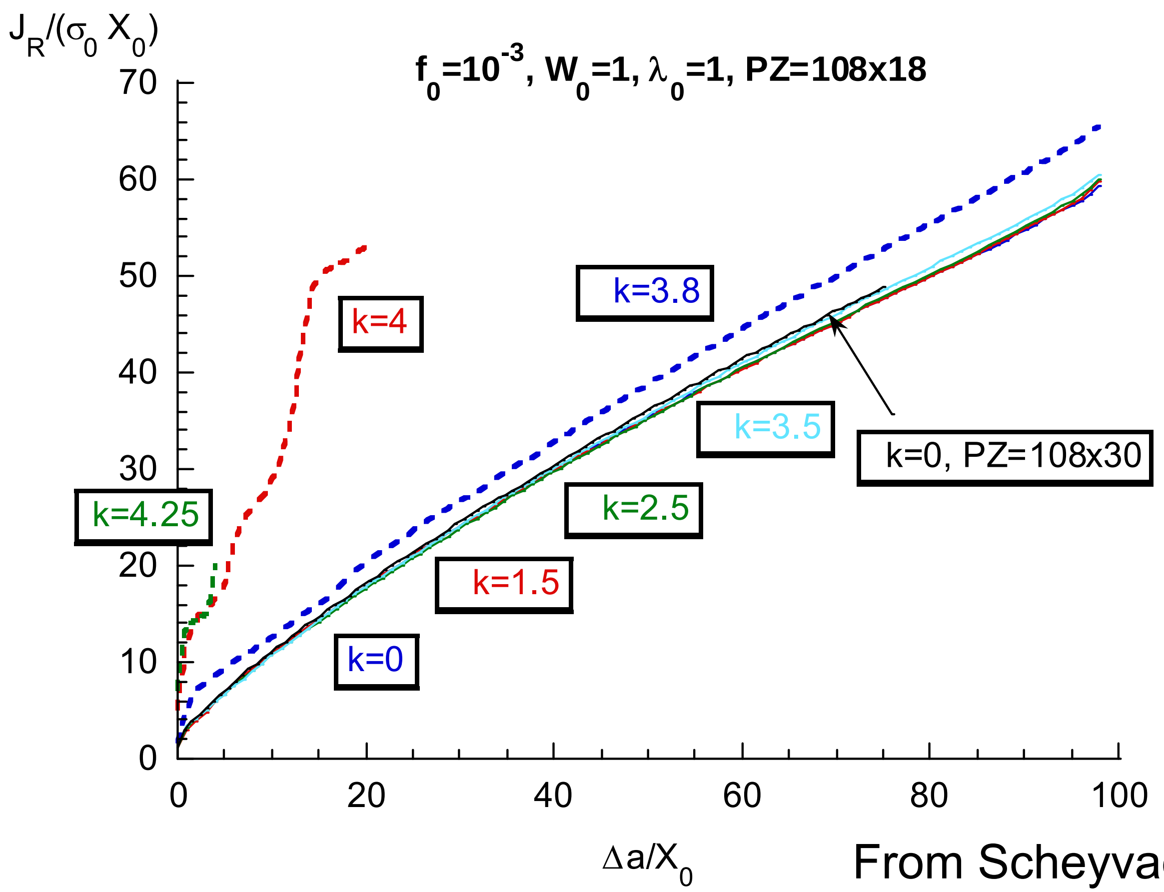
$$J_{Ic} = \sigma_0 X_0 F\left(n, \frac{\sigma_0}{E}, f_0, W_0, \lambda_0, \ldots\right)$$



# Plane strain ductile tearing resistance of ductile metallic alloys

## Relatively small impact of void nucleation stress on tearing modulus

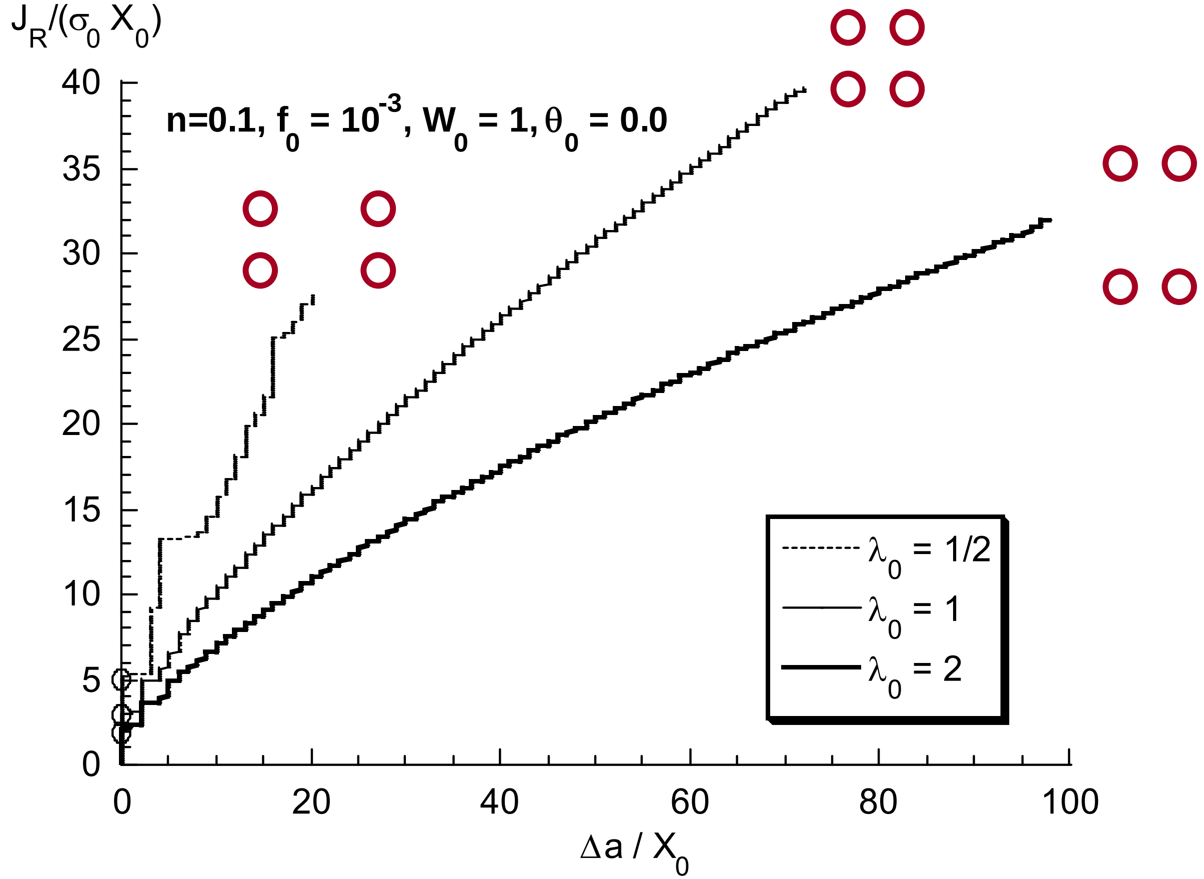
$$\sigma_{nucleation} = k\,\sigma_0$$



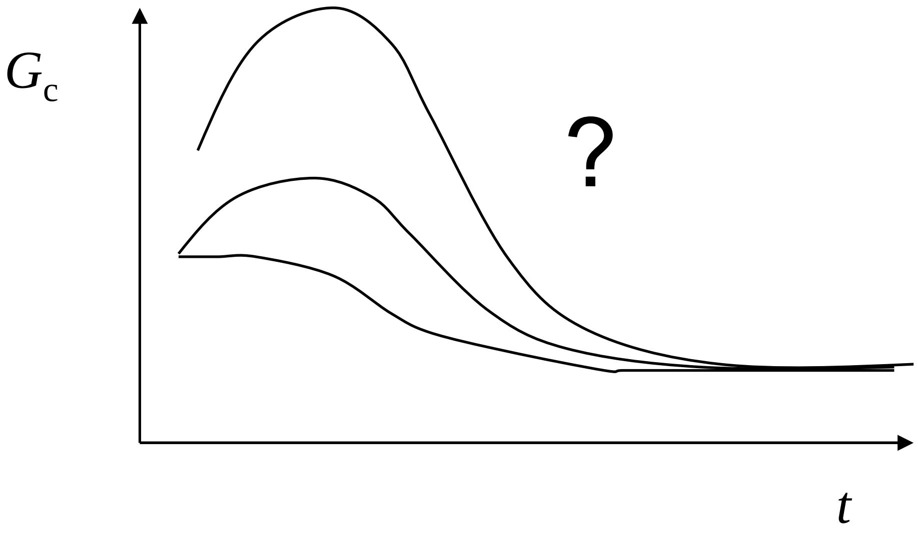

From Scheyvaerts Ph. D. UCL

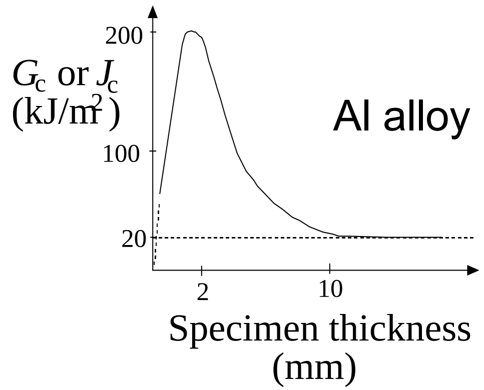
# Plane strain ductile tearing resistance of ductile metallic alloys

## Significant effect of void distribution



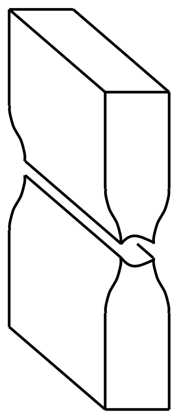
# Ductile cracking in thin components – many open questions



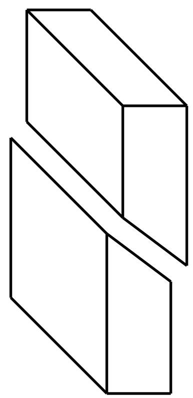

**or**

Mode I cracking



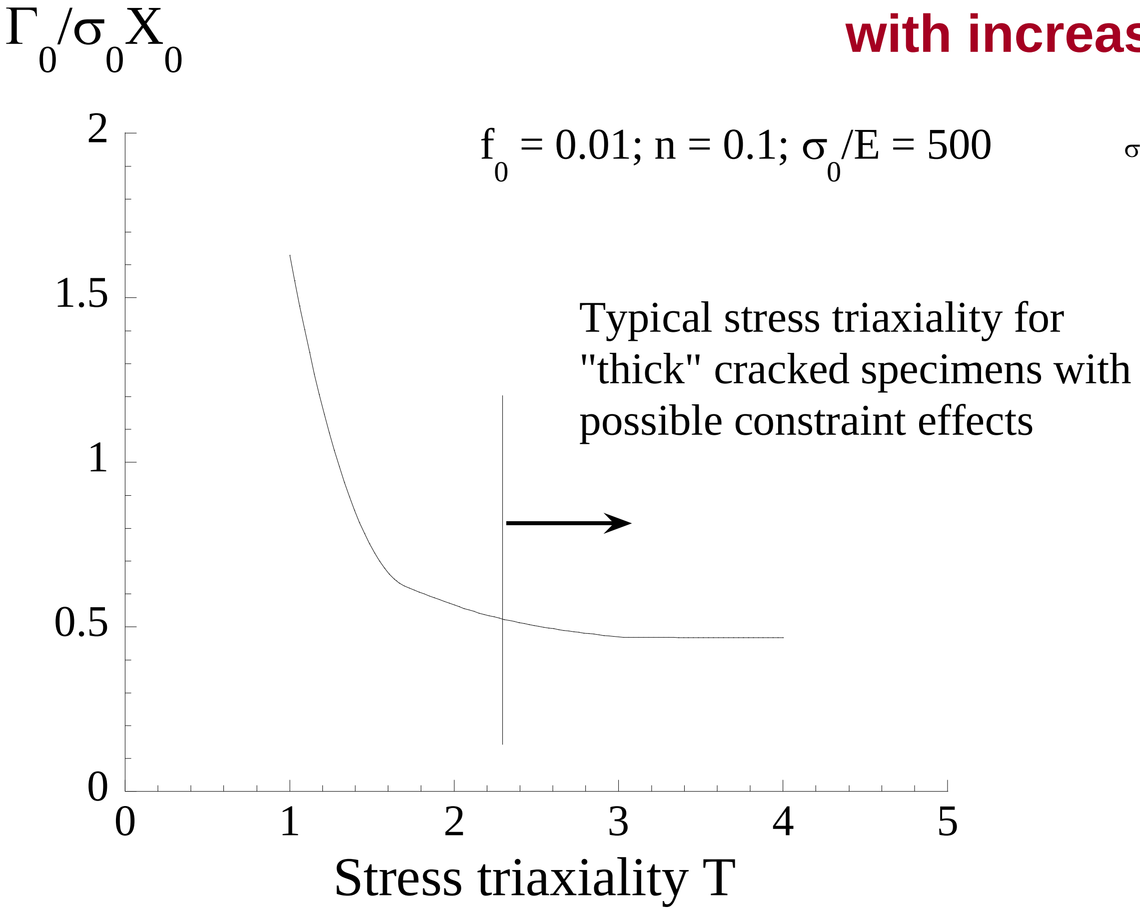
?

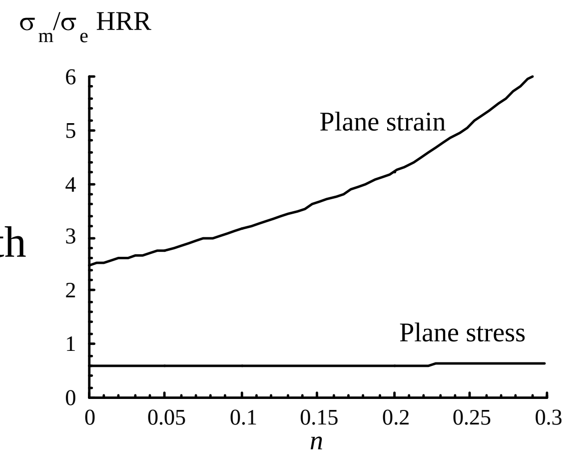
Mixed mode I-III cracking

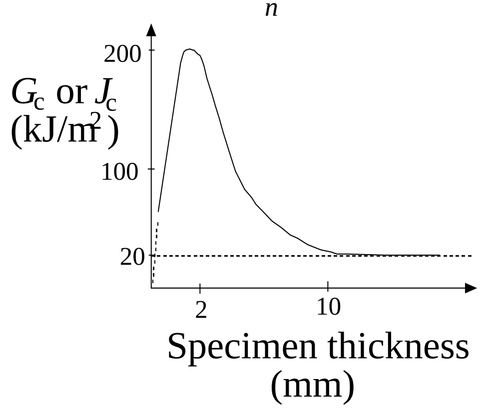


# Analytical « first order » model – explains why fracture toughness is larger in plane stress compared to plane strain

**But, it does not explain why fracture toughness would start first increasing with increasing thickness**



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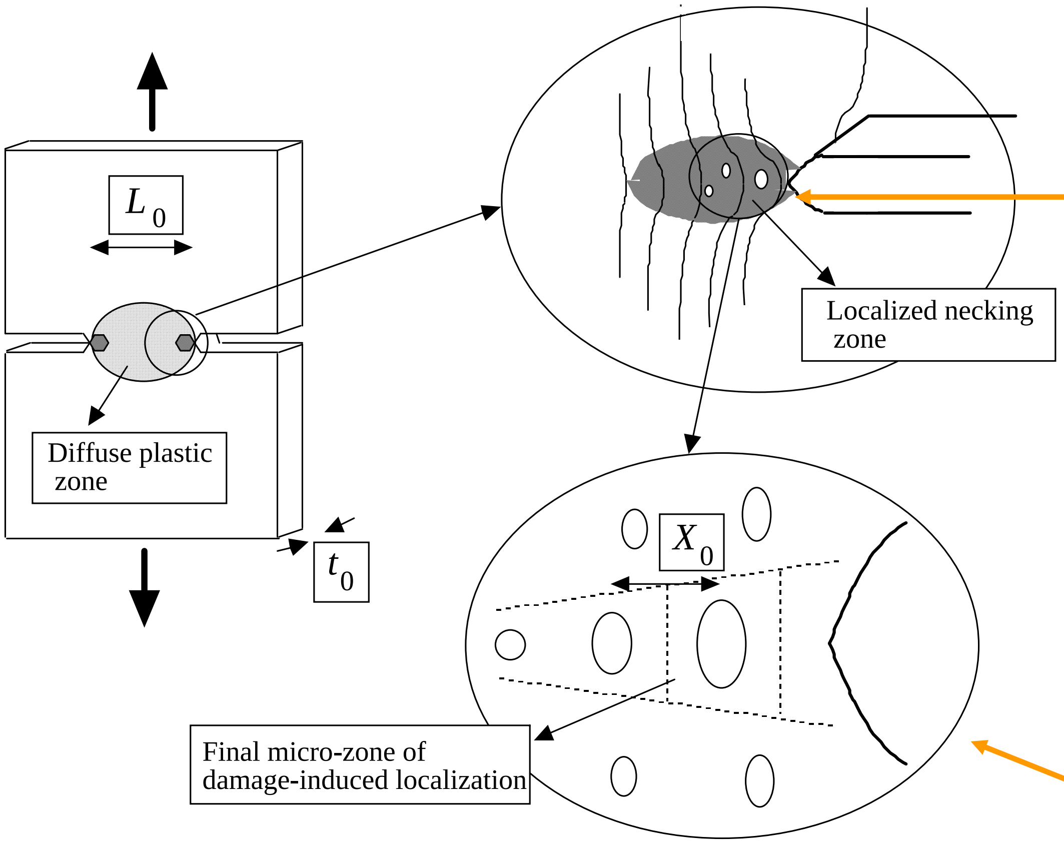
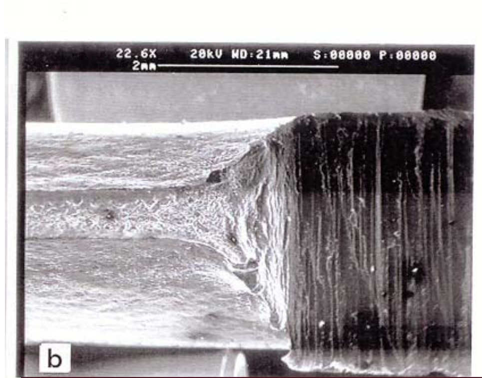
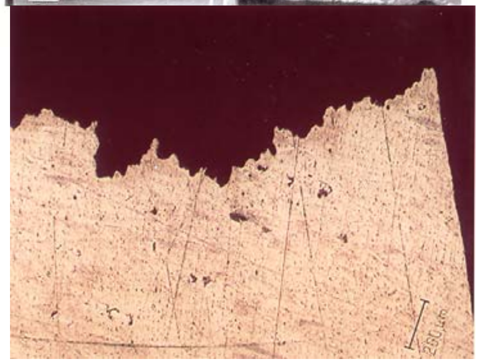
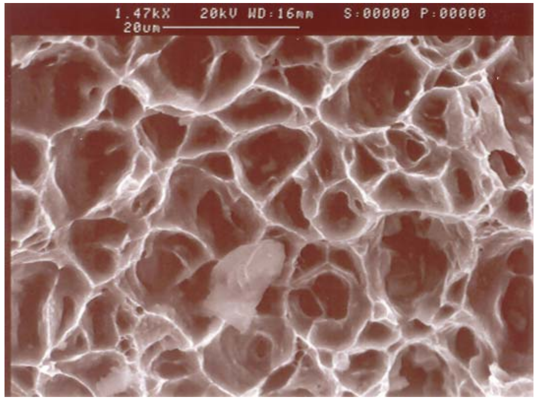
# Additional effect in thin plates: crack tip necking



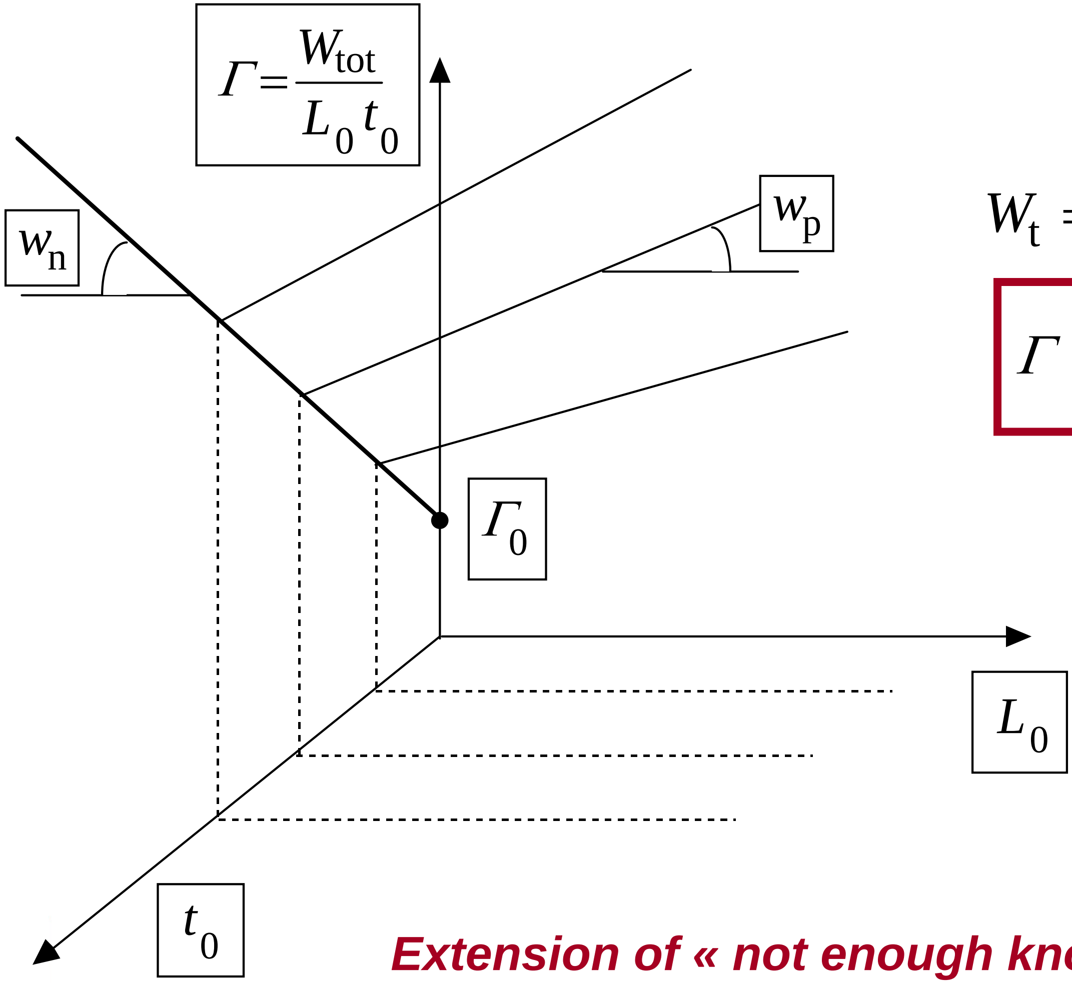





In thick plates $\Gamma(\Delta a) = \Gamma_0(\Delta a) + \Gamma_p(\Delta a)$

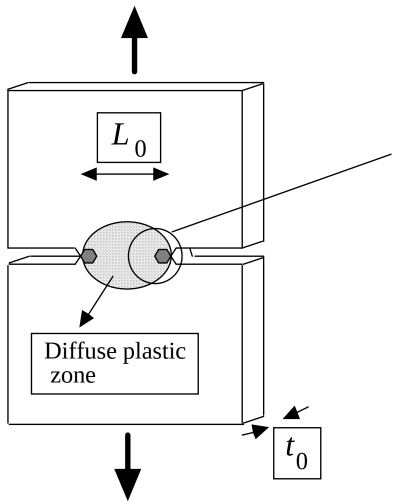
In thin plates $\Gamma(\Delta a) = \Gamma_0(\Delta a) + \Gamma_n(\Delta a) + \Gamma_p(\Delta a)$




In DENT panels

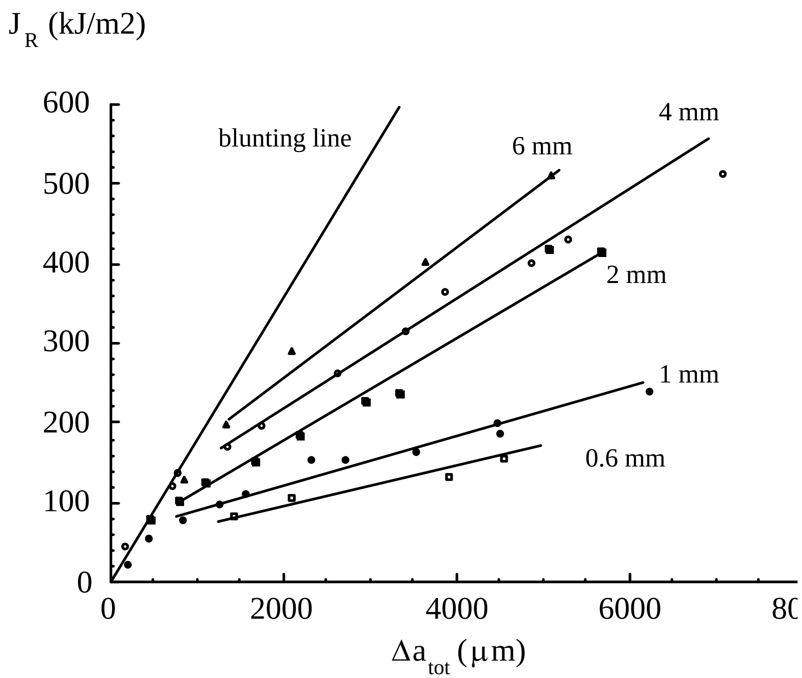
$$W_t = \Gamma_0 L_0 t_0 + w_n \alpha t_0^2 L_0 + w_p \beta t_0 L_0^2$$

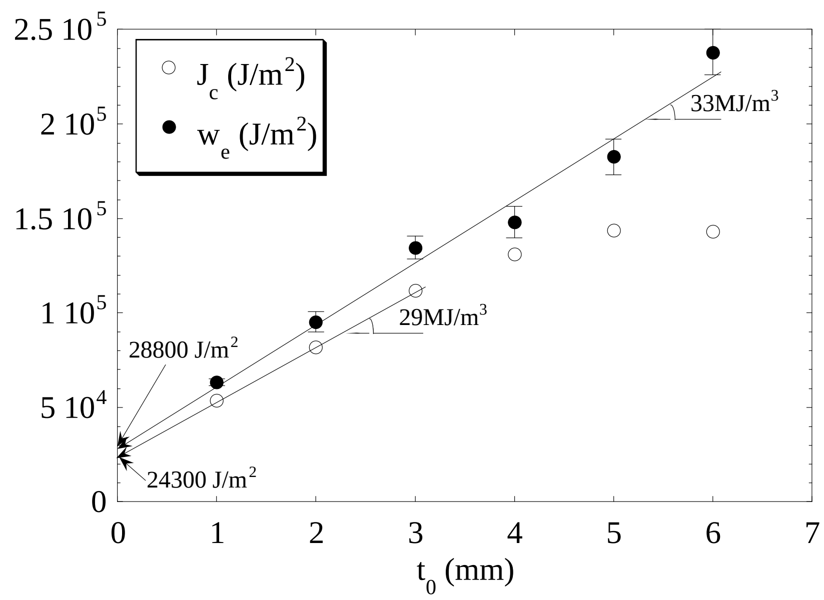
$$\Gamma = \Gamma_0 + w_n \alpha t_0 + w_p \beta L_0$$




***Extension of « not enough known in metallurgy » essential work of fracture method***

# Ductile tearing of thin 6082-O aluminium plates




Pardoen, T., Marchal, Y., Delannay, F., 1999. Thickness dependence of cracking initiation criteria in thin aluminum plates, J. Mech. Phys. Solids 47, 2093-2123.

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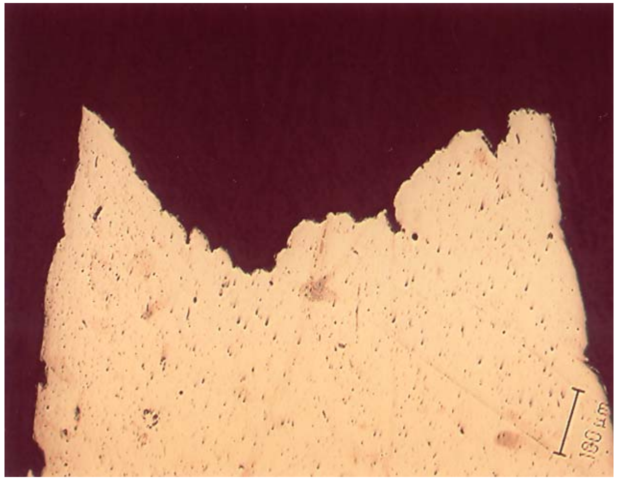
# *Same observations for a wide range of metals !!!*

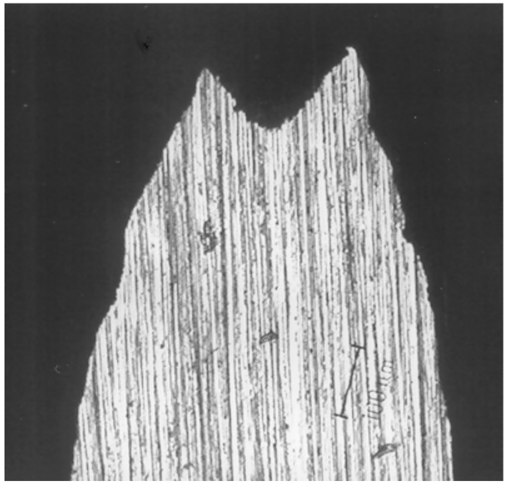
| Materials | Thickness (mm) | Homog. | $E$ (GPa) | $\sigma_0$ (MPa) | $n$ (Swift) | $k$ (Swift) |
|---|---|---|---|---|---|---|
| Stainless steel A316L | 0.65 to 3 | yes | 210 | 310 | 0.48 | 25 |
| Al 6082-O | 0.6 to 6 | yes | 70 | 50 | 0.26 | 265 |
| Brass annealed | 0.9 to 2 | yes | 110 | 100 | 0.6 | 33 |
| Al NS4 // RD | 0.57 to 1.5 | yes | 70 | 140 | 0.17 | 159 |
| Zinc // RD | 0.6 to 1.3 | yes | 61 | 100 | 0.15 | 118 |
| Lead | 0.8 to 1.8 | yes | 16 | 7 | 0.25 | 290 |
| Bronze annealed | 0.54 to 1.2 | yes | 100 | 120 | 0.51 | 38 |
| Bronze ⊥ RD | 0.54 to 1.2 | yes | 100 | 400 | 0.01 ? | ? |
| Bronze // RD | 0.54 to 1.2 | yes | 100 | 410 | 0.015 ? | ? |
| Brass ⊥ RD | 0.9 to 2 | no | 110 | (240) | (0.25) | |
| Brass // RD | 0.9 to 2 | no | 110 | (210) | (0.32) | |
| Al NS4 annealed | 0.57 to 1.5 | no | 70 | (80) | (0.2) | |
| Al NS 4 ⊥ RD | 0.57 to 1.5 | no | 70 | (150) | (0.14) | |
| Mild steel ⊥ RD | 0.87 to 1.5 | no | 210 | (240) | (0.17) | |
| Mild steel // RD | 0.79 to 1.5 | no | 210 | (220) | (0.17) | |
| Zinc ⊥ RD | 1.3 | yes | 86 | 140 | 0.08 | |

Pardoen, T., et al. 2004 . J. Mech. Phys. Solids 52, 423.

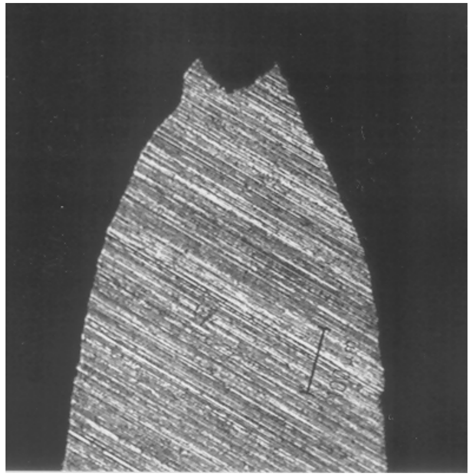
# *Cup & cup fracture*



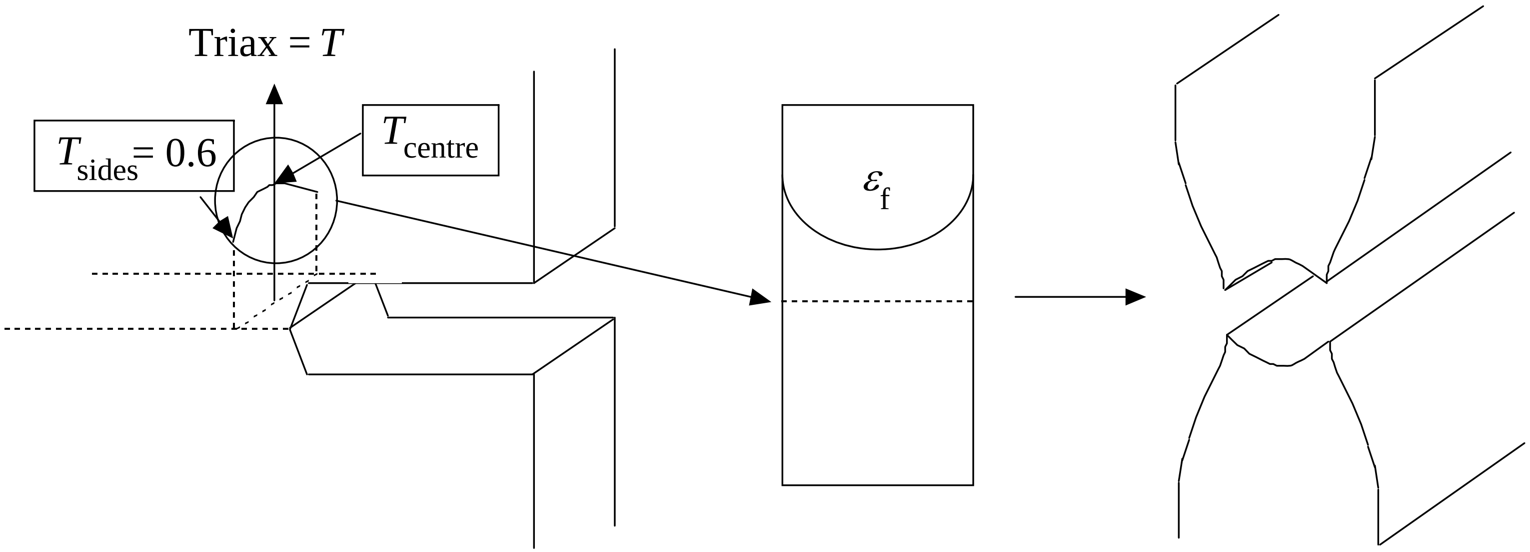
A316L



Brass Annealed



Al NS4



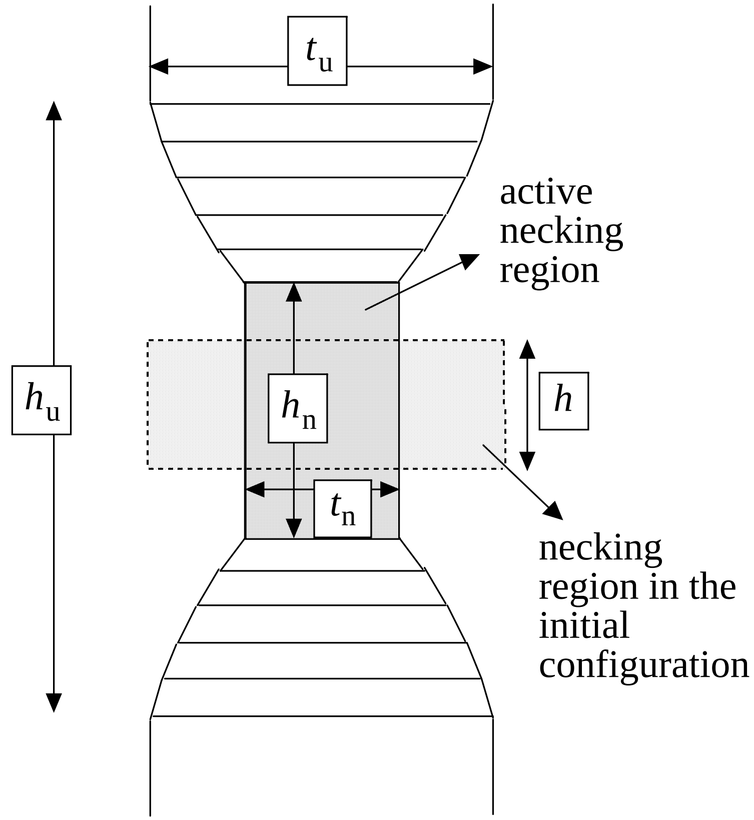
# Model for the work of necking

$$\frac{\Gamma_n}{\sigma_0} = F\left\lfloor n, k, \frac{\sigma_0}{E}, \nu, \varepsilon_f \right\rfloor$$

$$W_n = 2 l_0 t_0 \int_0^{h_n(\bar{\varepsilon}_u)/2} \int_{\bar{\varepsilon}_u}^{\bar{\varepsilon}_n^{max}(h)} \bar{\sigma} d\bar{\varepsilon} dh$$

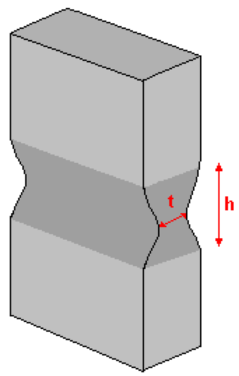
$$\Gamma_n = 2 \int_0^{h_{u0}/2} \int_{\bar{\varepsilon}_u}^{\bar{\varepsilon}_n^{max}(h)} \bar{\sigma} d\bar{\varepsilon} dh$$

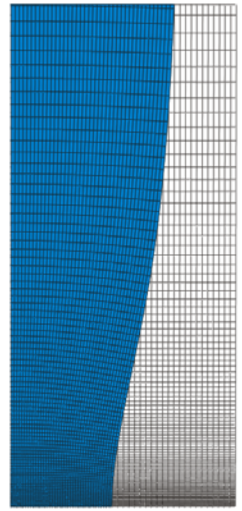
Assumption of plane strain tension




$$\bar{\varepsilon}_u = \frac{2nk - \sqrt{3}}{\sqrt{3}k}$$

# Model for the work of fracture

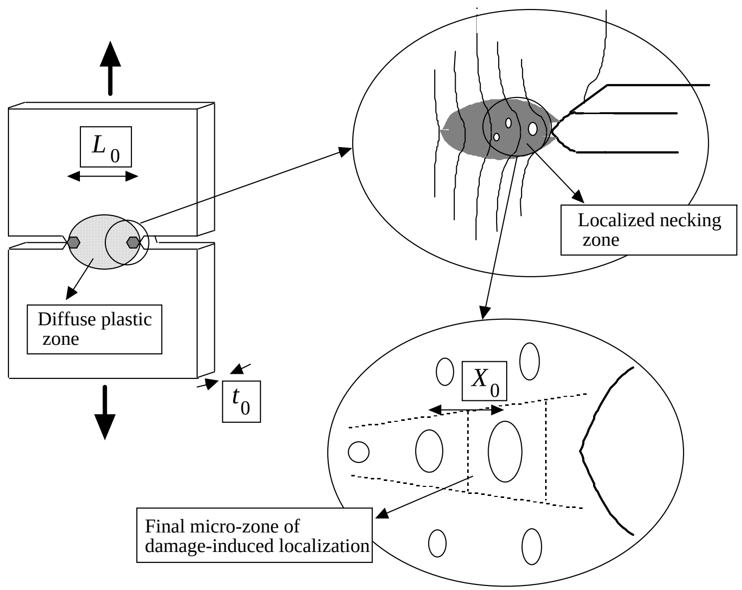


Extended Gurson model (Gologanu + Thomason)

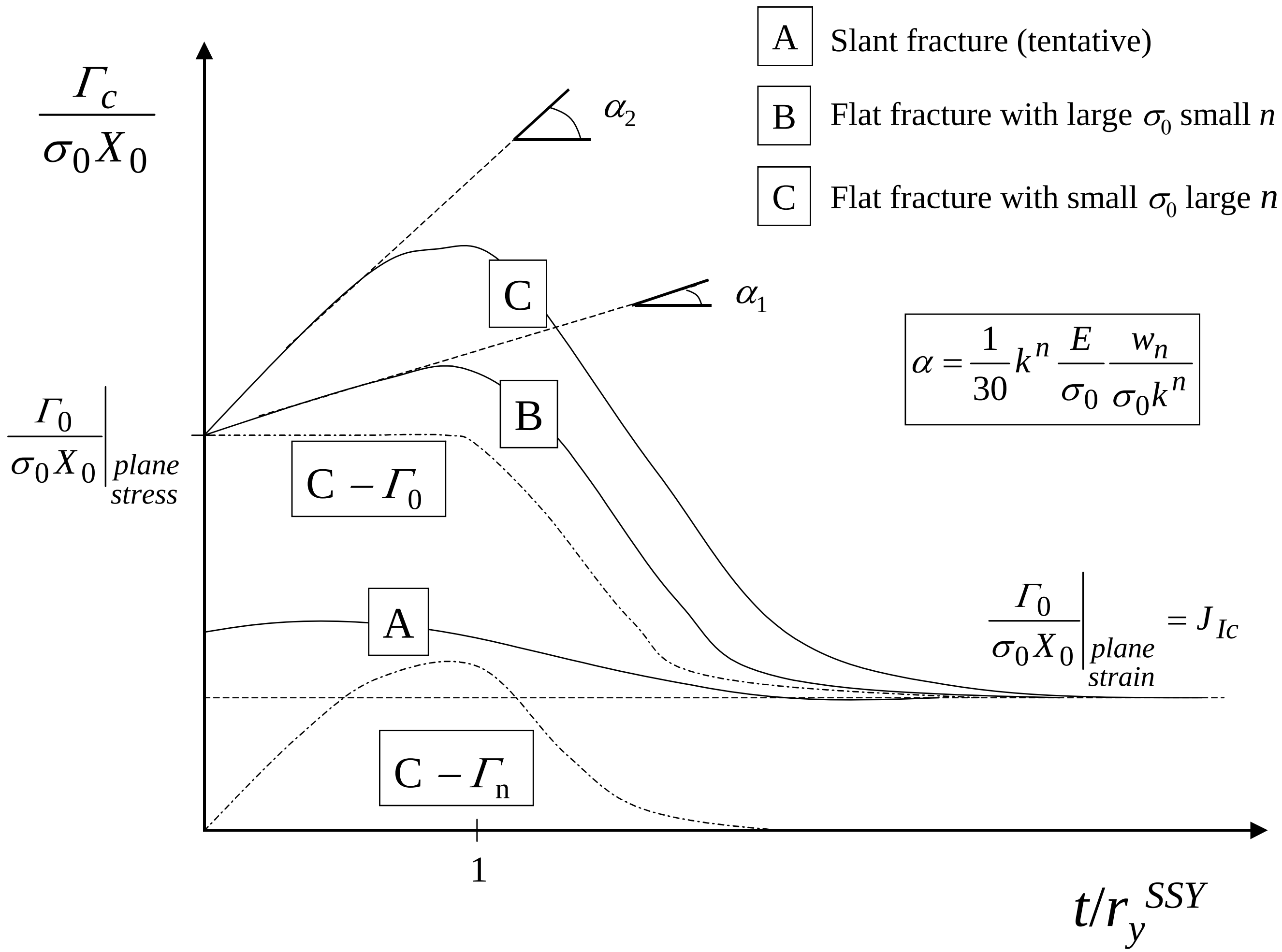
$$\Phi_{growth} \equiv \frac{C}{\bar{\sigma}^2}\left\|\mathbf{s} + \eta\sigma_{hg}\mathbf{X}\right\|^2 + 2q(g+1)(g+f)\cosh\left(\kappa\frac{\sigma_{hg}}{\bar{\sigma}}\right) - (g+1)^2 - q^2(g+f)^2 = 0$$

$$\Phi_{coalescence} \equiv \frac{\sigma_e}{\bar{\sigma}} + \frac{3}{2}\frac{|\sigma_h|}{\bar{\sigma}} - F(W,\chi) = 0$$

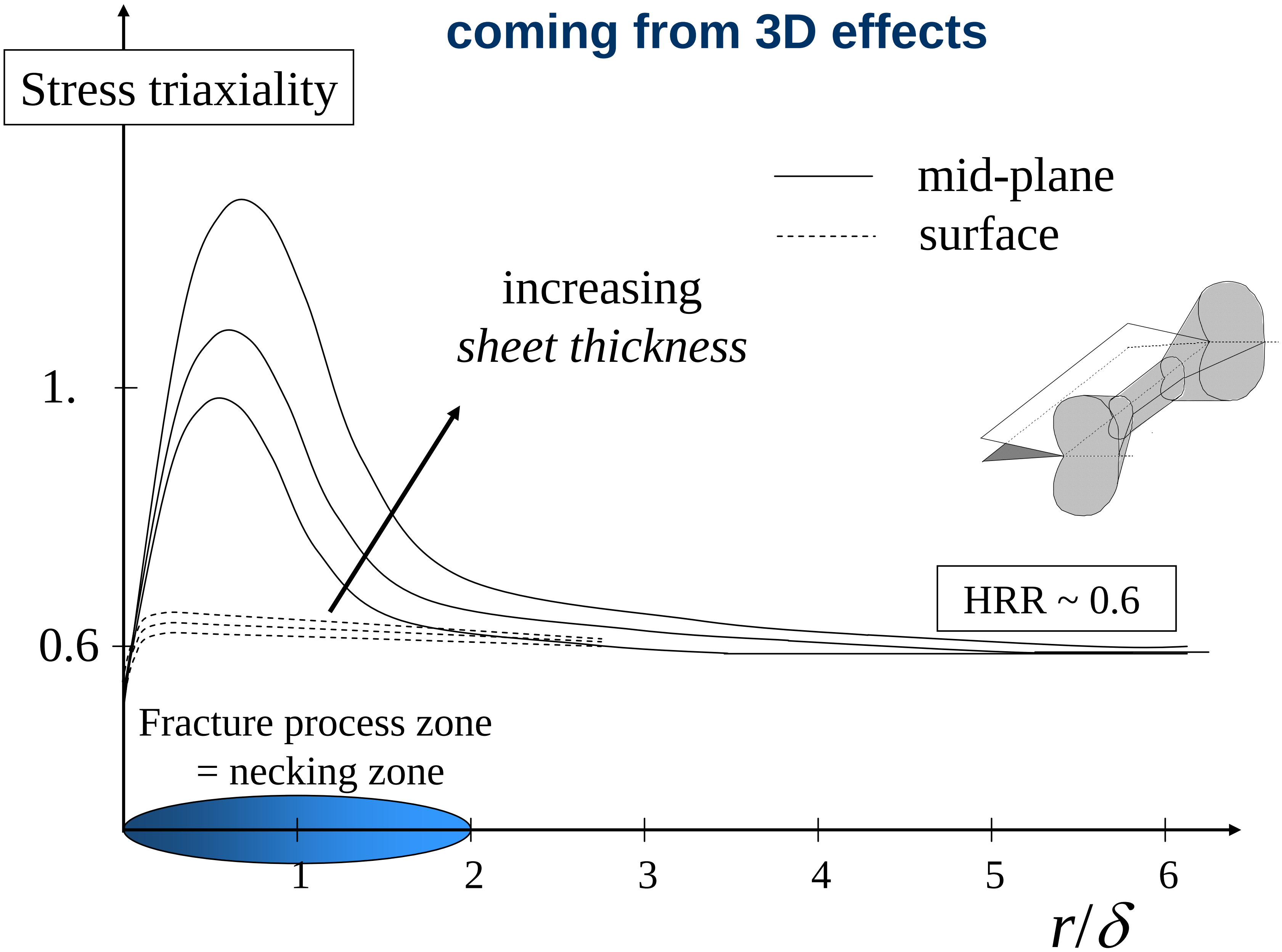



$$\boxed{\frac{\Gamma_0}{\sigma_0 X_0} = F\left(\frac{\sigma_0}{E}, n, f_0, W_0, \lambda_0\right)}$$

# Combining the work of necking and work of fracture

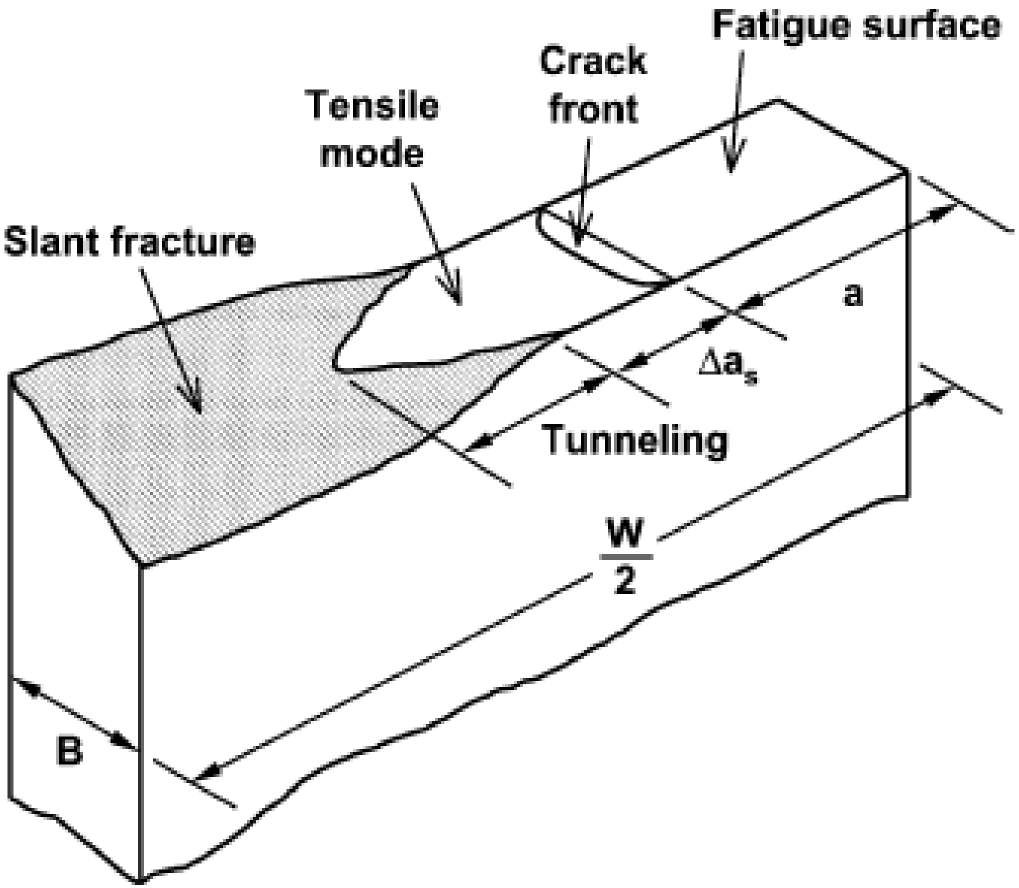


# Additional complexity coming from 3D effects



# Additional complexity coming from 3D effects

**Crack tunneling**
**Transition into slant fracture**



# Plan de la présentation

**1. Introduction**
Pourquoi la mécanique de la rupture ?

**2. Bases de la mécanique linéaire élastique de la rupture**
Notions de champs en K, G, taille de zone plastique $K_{IC}$, $G_{IC}$, validité

**3. Bases de la mécanique élastoplastique de la rupture**
Notions de champs de HRR, J, CTOD, JR curve, grandes déformations

**4. Lien entre la ténacité et la microstructure/mécanismes dans les métaux**
A. Rupture fragile – modèle RKR
B. Rupture ductile – modèle de croissance-coalescence de cavités
C. Cas de tôles minces métalliques

**5. Limites de la mécanique de la rupture**
Effets de géométrie, paramètre unique, invalidité, complémentarité avec approche par micromécanique de l'endommagement

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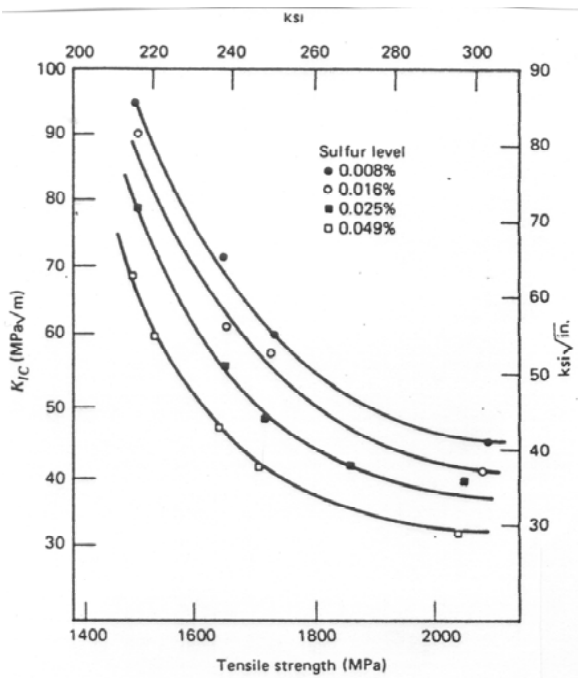
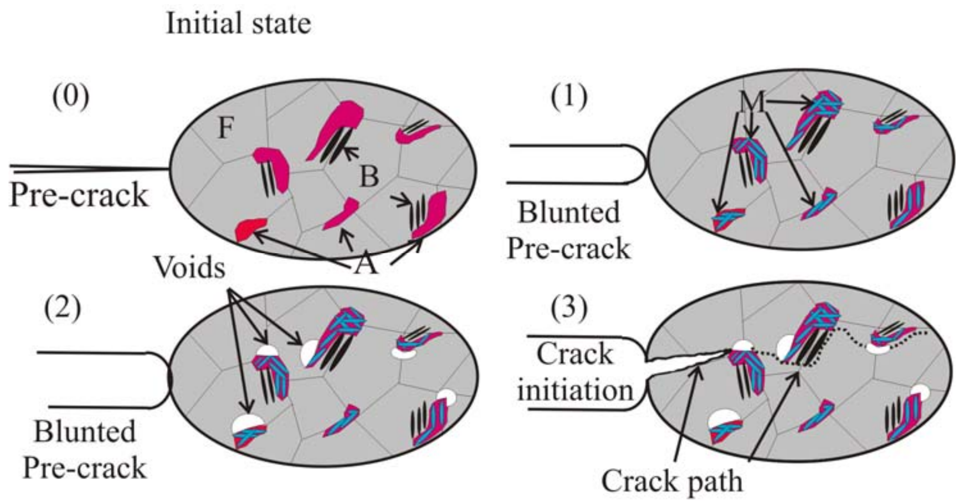
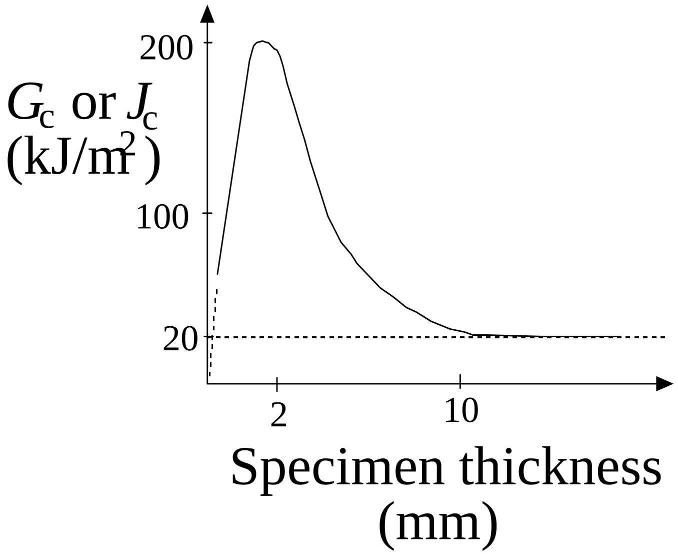
# Why is fracture mechanics not sufficient ?
# or
# Why do we need to link it with physical mechanisms and address a local (or micromechanics) approach of ductile fracture ?

***Knowing the fracture toughness (one number) – or tearing modulus (another number) = no info about how improving material structure***



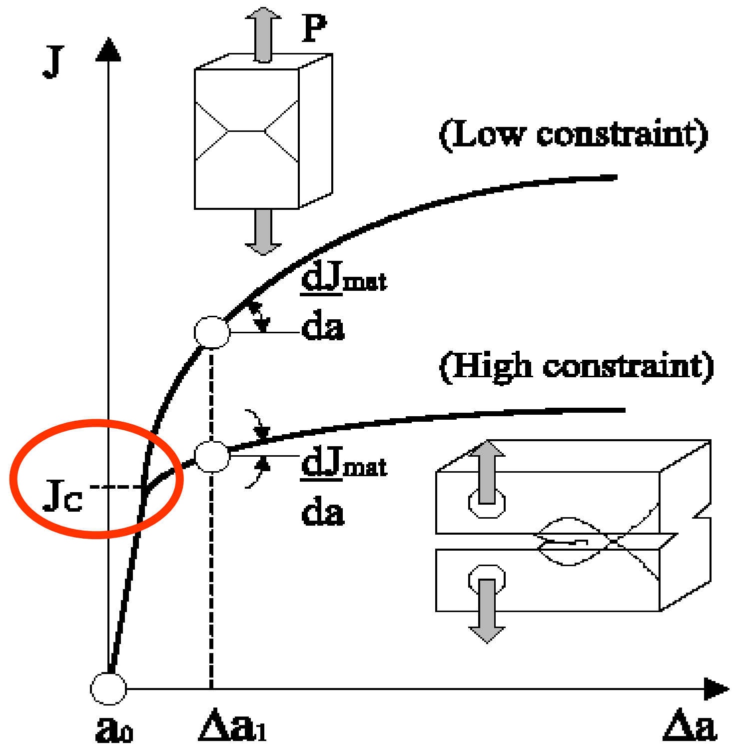



## *« Limitations » of fracture mechanics related to « constraint effects »*




Fracture toughness depends on plate thickness

Tearing resistance depends on geometry, dimensions and loading configuration




From X.K. Zhu, S.K. Jang / Engineering Fracture Mechanics 68 (2001) 285-301

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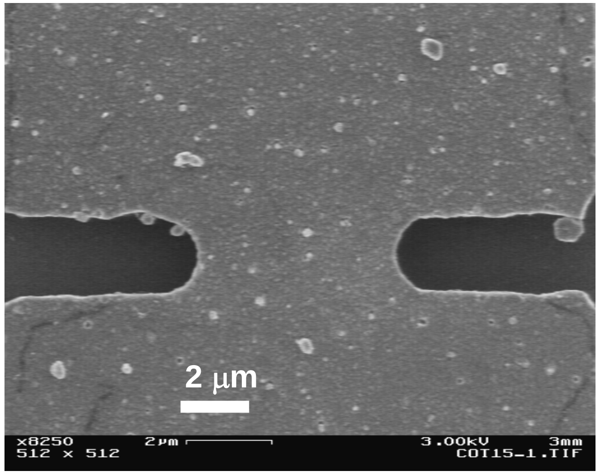
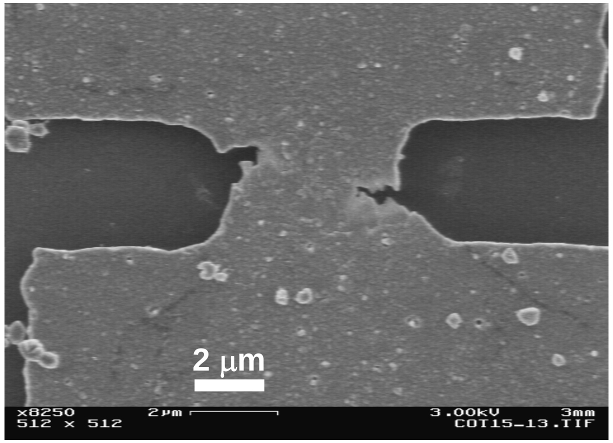
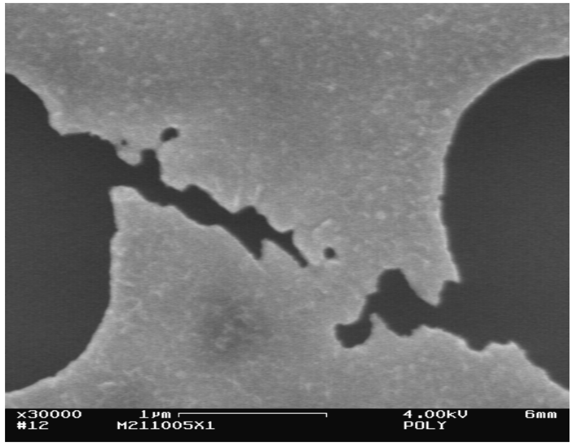
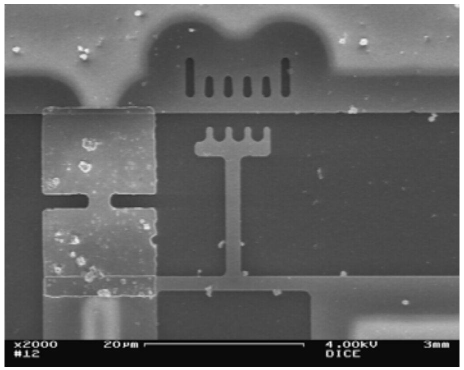
# One specific open question - Damage and fracture mechanisms in submicron thin films

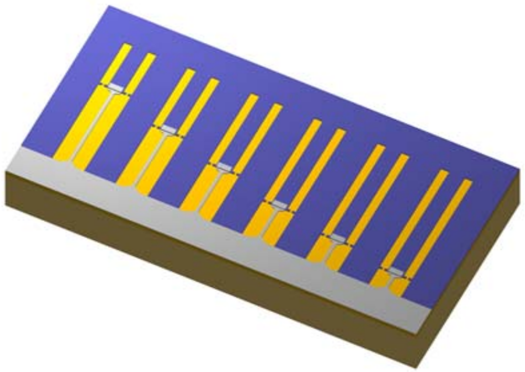


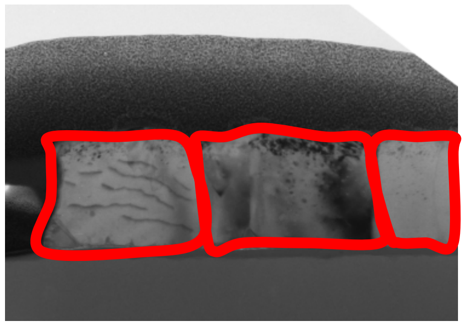




**250 nm thick Al films tested on chip – grain size = 200 nm – void nucleation at GB – heavily rate dependent – size much too small for applying fracture mechanics …**





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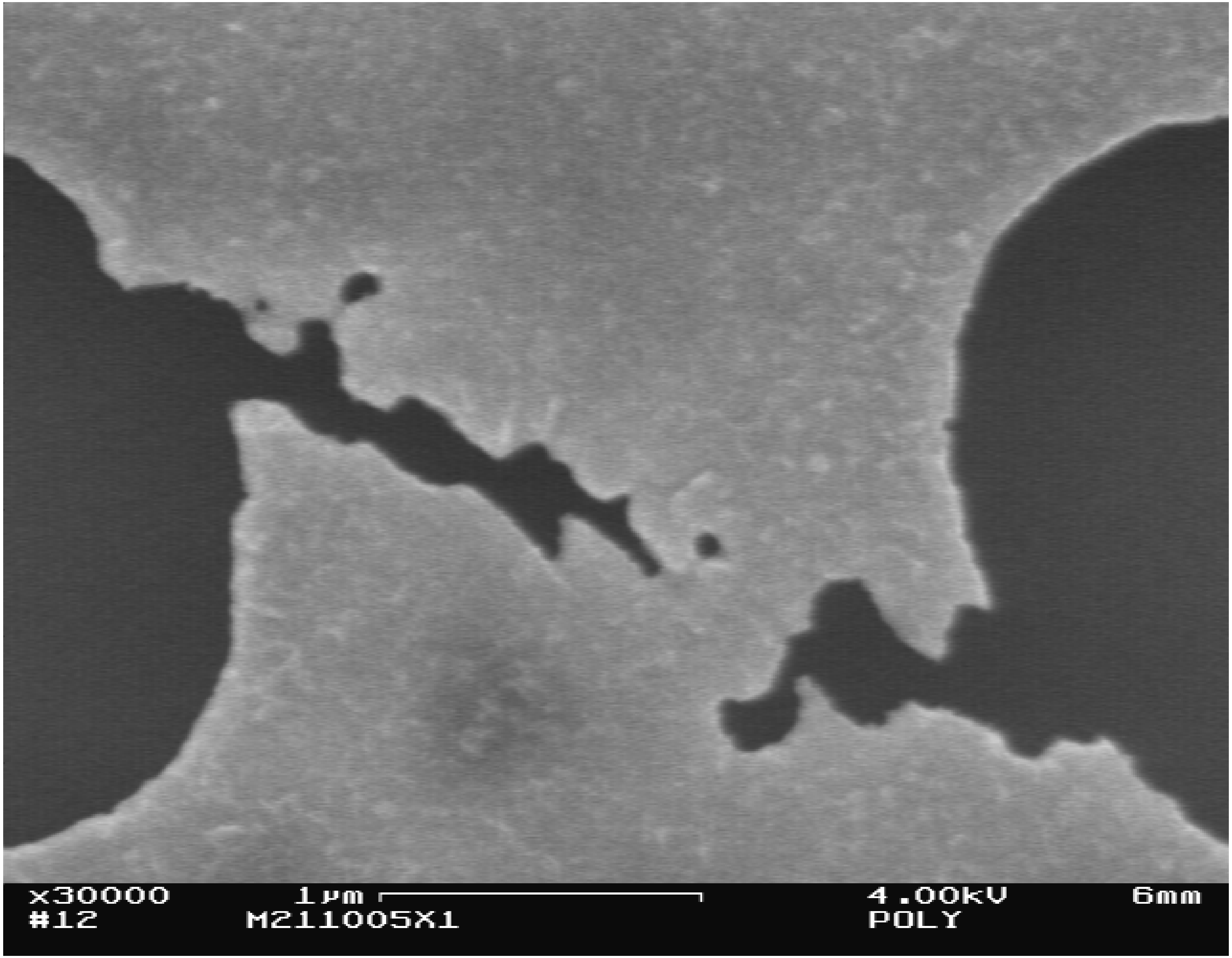

# Key topics

**« always combine fracture mechanics with microstructure informed micromechanics approaches/local approach »**

- Competition of failure modes (trans versus inter; brittle versus ductile, etc)

- Viscoplasticity effects on damage at the crack tip, especially in thin sheets

- Failure of thin metallic sheets remains a challenging topic

- Link between fracture toughness and micromechanical approaches remain insufficiently addressed

- A variety of emerging fields where fracture of metals is essential and metallurgists can play a key role : stretchable electronics, fracture in Li batteries, thin metallic coatings …