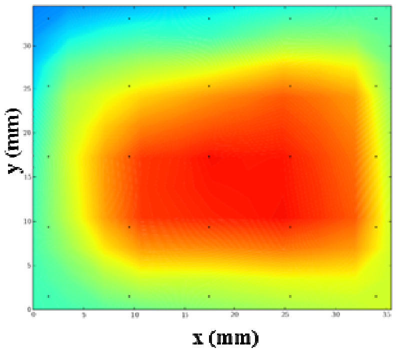
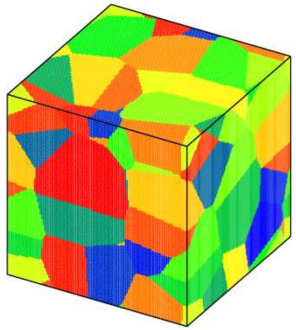
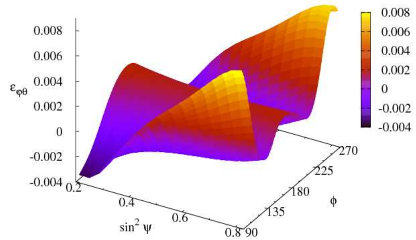
DE LA RECHERCHE À L'INDUSTRIE

RESIDUAL STRESSES







ANF Métallurgie Fondamentale | Vincent Klosek (CEA / DSM / IRAMIS / LLB)

23/10/2012

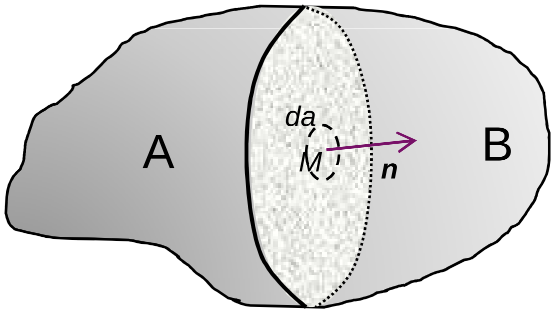
Residual Stresses ?

- Static multiaxial stresses within an isolated solid in mechanical equilibrium, neither subjected to external force nor to external moment

→ Result from the thermo-mechanical history of the considered material

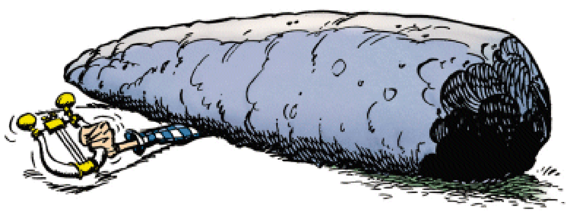
Notion of stress

- Modeling internal efforts → **2^{nd} order symmetric tensor**
- Cauchy stress tensor defined at a point: defines the linear application which determines the stress vector *T* for any facet through this point



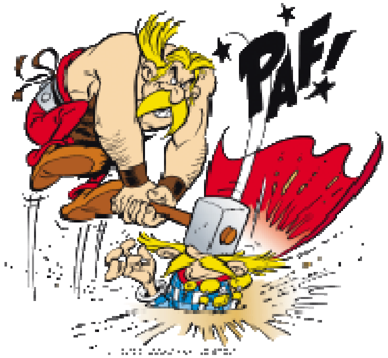
$$T(\mathrm{M}, n) = \boldsymbol{\sigma} \cdot n$$

- **σ traduces « contact actions » between material particles**



REMINDERS (?)

Notion of strain



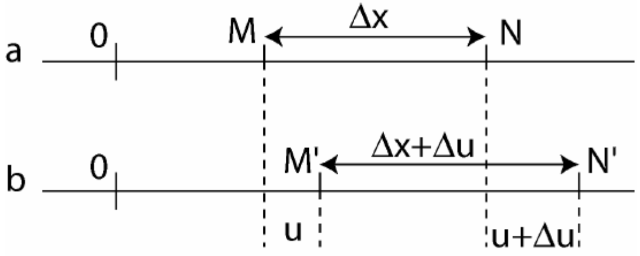
- What one can measure…
  (*one does not measure a force* → *one makes it work!*)

- **Strain tensor**:
  allows to express lengths and angles variations during a transformation

- Small Perturbations Hypothesis → **linearised tensor ε** :

$$\boldsymbol{\varepsilon}(X) = \frac{1}{2}(\nabla\xi(X) + {}^{t}\nabla\xi(X))$$

1D



$$\varepsilon = \lim_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}\right) = \left(\frac{du}{dx}\right)_M$$

Constitutive Laws

- Local relations between $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$ and T
- **Thermoelasticity** in the framework of infinitesimal strain theory (→ linearized relation):

$$\boldsymbol{\sigma} = \mathbb{A} : \boldsymbol{\varepsilon} - \mathbf{k}(T - T_0)$$

- Isotropic elastic material, isothermal transformation:

$$\sigma_{ij} = \lambda \cdot (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) \cdot \delta_{ij} + 2\mu\varepsilon_{ij}$$

Young modulus: $$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}$$

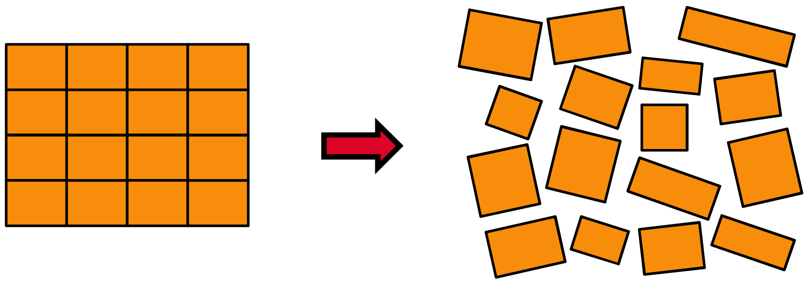
Poisson ratio: $$\nu = \frac{\lambda}{2(\lambda + \mu)}$$

Strain Incompatibility

- **Field ε must be compatible** (⇔ derived from a displacement field):

$$\mathrm{rot}^{g}(\mathrm{rot}^{d}\boldsymbol{\varepsilon}) = 0$$

- What if not ? Loss of continuity !



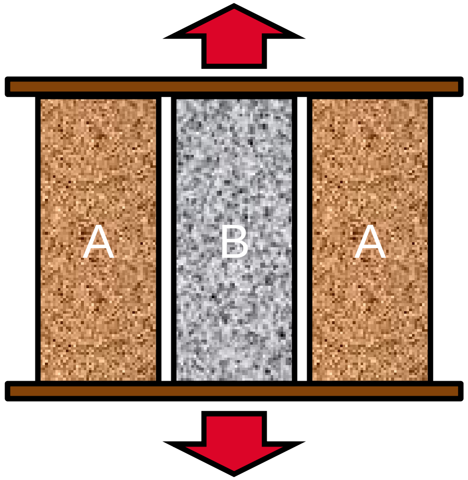
- If strain field not compatible BUT continuity remains
  → ∃ additional elastic field so that the total strain field is compatible!

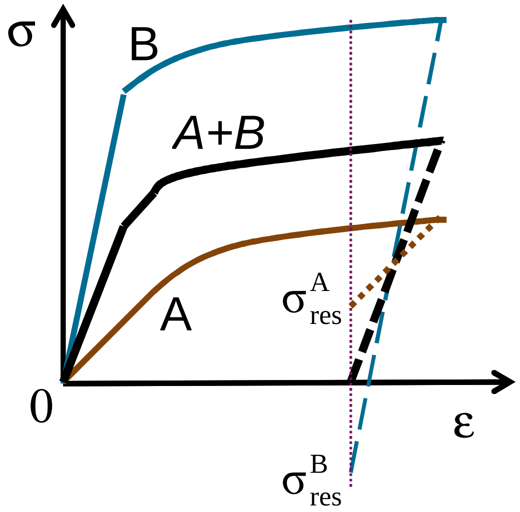
$$\varepsilon^{\text{total}} = \varepsilon^{\text{stress free}} + \varepsilon^{\text{elastic accommodation}}$$

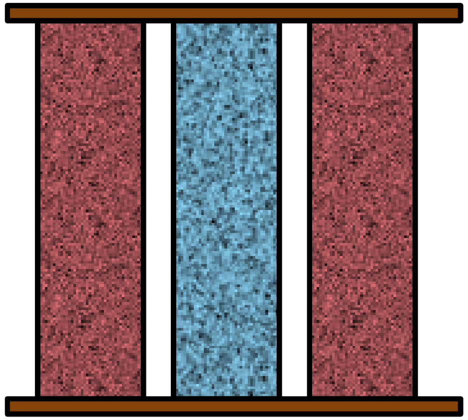
→ **Internal stress associated with elastic accommodation**

Strain Incompatibility

- Application of a compatible strain field to a heterogeneous material:



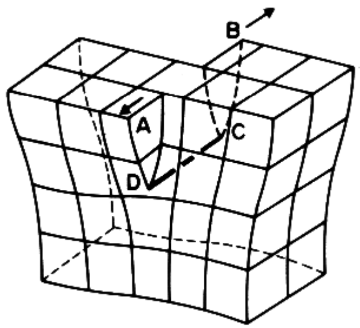


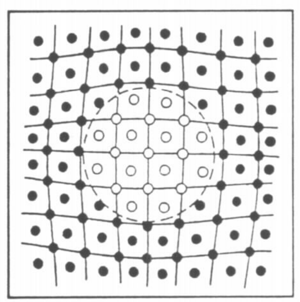


→ **RESIDUAL STRESSES**

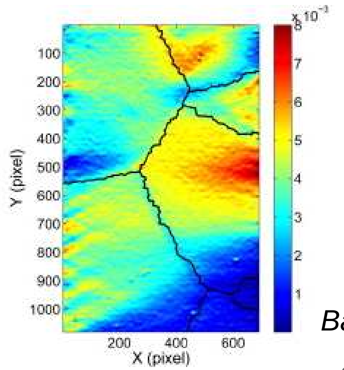
Main sources → various scales

- Atomic scale: point defects (very local influence)
- Single crystal scale: dislocations, precipitates, etc…





- Polycrystal scale:
  - → Plastic strain incompatibilities
  - → phase transformations
  - → thermal strains…



Badulescu et al. (2011)

RESIDUAL STRESSES ?

Various scales…

$$\underline{\underline{\sigma}}^{res}(\mathbf{X}) = \underline{\underline{\sigma}}^{I}(\mathbf{X}) + \underline{\underline{\sigma}}^{II}(\mathbf{X}) + \underline{\underline{\sigma}}^{III}(\mathbf{X})$$

- 1st order stresses: in equilibrium at the whole sample scale (→ scale considered by engineers for structures calculations)

$$\underline{\underline{\sigma}}^{I}(\mathbf{X}) = \frac{1}{V}\int_{V} \underline{\underline{\sigma}}^{res}(\mathbf{X})$$

- 2nd order stresses: in equilibrium at the scale of a group of crystallites

$$\underline{\underline{\sigma}}^{II}(\mathbf{X}) = \frac{1}{v}\int_{v} \underline{\underline{\sigma}}^{res}(\mathbf{X})$$

- 3rd order stresses: in equilibrium at the scale of a crystallite
→ Defects of crystallographic lattice (dislocations, precipitates, vacancies, grain boundaries, etc…)

$$\underline{\underline{\sigma}}^{III}(\mathbf{X}) = \underline{\underline{\sigma}}^{res}(\mathbf{X}) - (\underline{\underline{\sigma}}^{I}(\mathbf{X}) + \underline{\underline{\sigma}}^{II}(\mathbf{X}))$$

RESIDUAL STRESSES ?

Why evaluating them is so important?...

- Strongly **influence the mechanical behaviour of a component**

  - **harmful**: premature fracture (fatigue, cracking, etc…)
  - **favourable**: example of prestress treatments (shot peening, etc…)

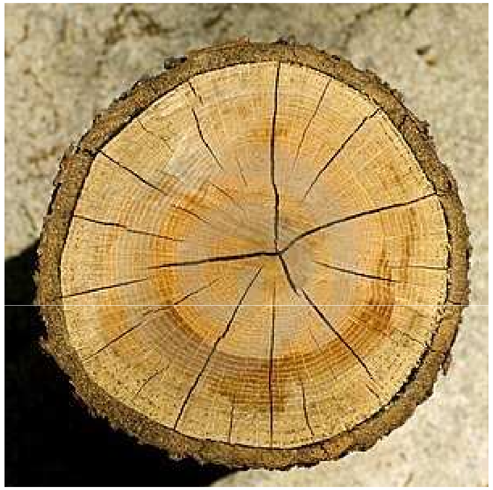
  → Significant effects on performance, safety, reliabillity of a component, a structure.
  → Improvement of the processes (thermal or mechanical treatment required ?...)

- At a more fundamental point of view:

  → Understanding of the physical deformation mechanisms, and how they interact

RESIDUAL STRESSES ?

Examples of significant manifestations



Drying of wood

Stress-induced corrosion

Silver Bridge (WV, USA), 1967

DETERMINATION METHODS

Experimental determination of elastic strains

- **Destructive or semi-destructive methods**

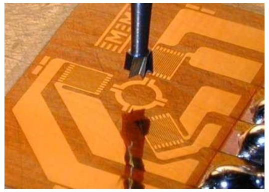
- **Non destructive methods**

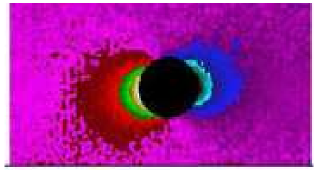
DETERMINATION METHODS

(semi-) destructive methods

- **<u>Principle:</u> some matter is removed → mechanical equilibrium is affected → system tends to reach a new equilibrium → it changes its shape**

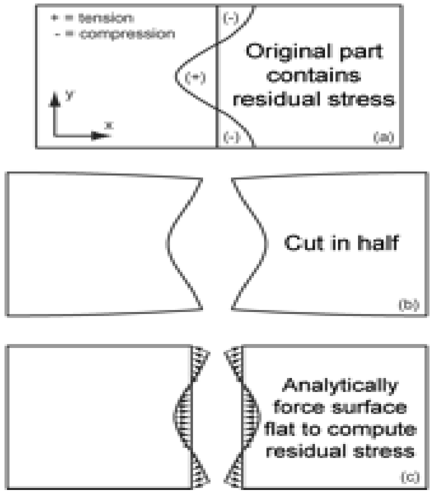
- **<u>Hole drilling method</u>** (incremental or not) or **<u>ring core method</u>**





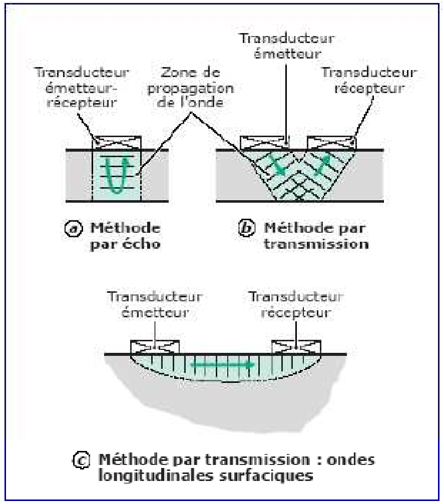
plane stress approximation, isotropic elasticity law, empirical coefs to determine…

- **<u>Contour method</u>**



DETERMINATION METHODS

Non destructive methods



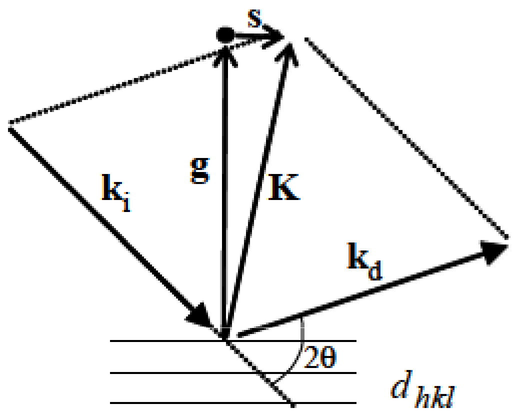
- **<u>Ultrasonic technique</u>**
  ⇔ dependence of the propagation velocity of ultrasonic waves on stress state
  (non linear elasticity, use of acoustoelastic coefficients)

- **<u>Barkhausen noise</u>** (ferromagnetic materials)
  ⇔ Discontinuous motion of Bloch walls
  stress ⇔ inverse magnetostrictive effect
  → technique very sensitive to microstructural defects (…tricky to isolate contributions)

- **<u>Raman spectrometry</u>**
  ⇔ dependence of optic phonon frequencies on stress state

- **<u>Diffraction</u>**
  Crystal lattice is used as a strain gauge…

Theory of diffraction by distorted crystals *(Krivoglaz, 1969)*

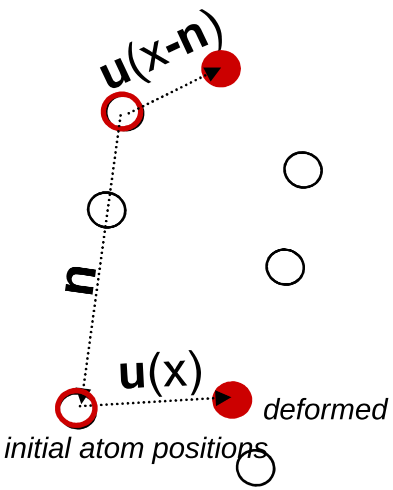


Diffraction vector $\mathbf{K} = \mathbf{g} + \mathbf{s}$

($\mathbf{g} \in$ *reciprocal lattice)*

Bragg: $I \neq 0 \Leftrightarrow \mathbf{s} = \mathbf{0}$

(non distorted perfect crystal)



If ∃ displacement field **u(x)**

Scattered intensity (normalised):

$$I(\mathbf{K}) = \iint e^{2\pi i \mathbf{n}\cdot\mathbf{s}} \cdot e^{2\pi i \Delta\mathbf{u}\cdot\mathbf{K}} d\mathbf{x} d\mathbf{n}$$

($\mathbf{n} \in$ *direct lattice)*

$$\Delta\mathbf{u}(\mathbf{x},\mathbf{n}) = \mathbf{u}(\mathbf{x}) - \mathbf{u}(\mathbf{x}-\mathbf{n})$$

DIFFRACTION METHODS

-Intensity distribution *I*(**K**) directly related to the projection of the relative displacement between atoms $\Delta\mathbf{u}$ along **K**

→ **Directional measurement** !

$$\varepsilon_{KK}(\mathbf{x}) = \frac{1}{K^2}\mathbf{K}\cdot\underline{\underline{\boldsymbol{\varepsilon}}}(\mathbf{x})\cdot\mathbf{K} = \lim_{n\to 0}\frac{\Delta\mathbf{u}\cdot\mathbf{K}}{nK}$$

- Periodicity breaking → **diffraction peak broadening** for **K** || **g**

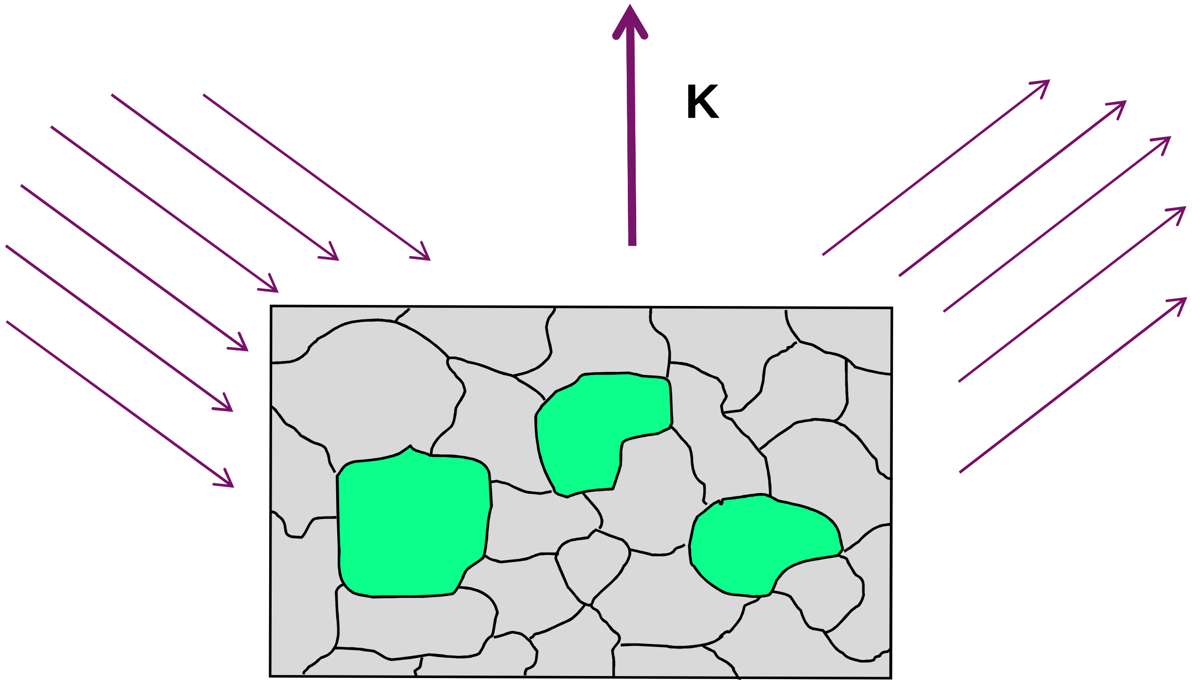
- Homogeneous deformation → **diffraction peak shift**

(*back to Bragg law!*)

- Diffraction technique only sensitive to elastic strains !

Selectivity of diffraction methods:

→ Strain is measured for grains in diffraction condition only !!!



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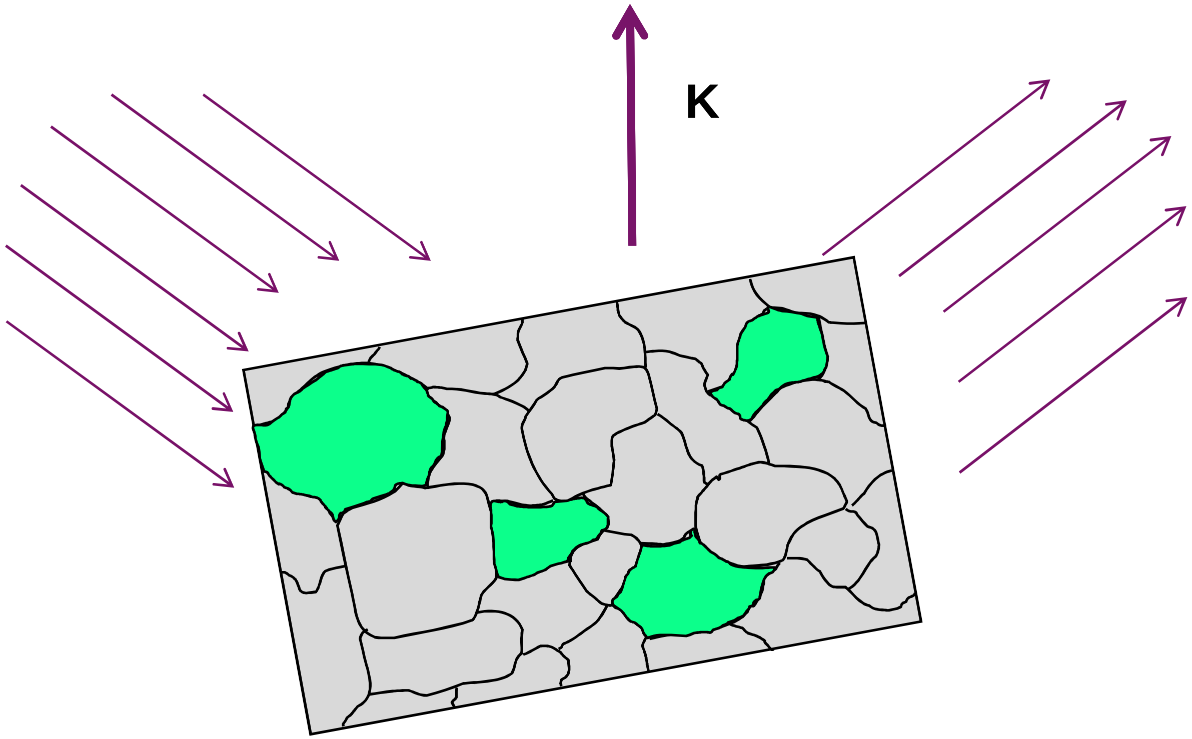
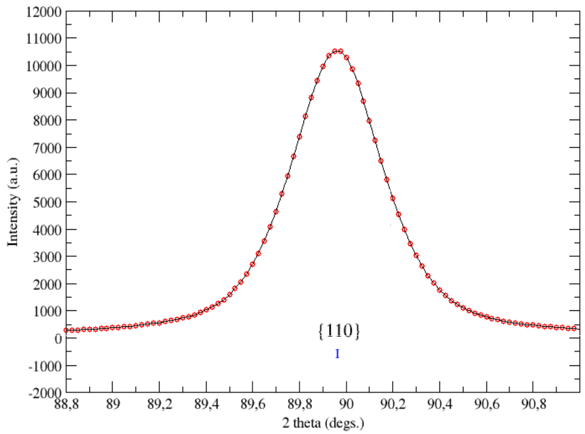


What do we measure ?...



1st order moment

$$\mu^{(1)} = \int_{-\infty}^{+\infty} s \cdot I(s)ds$$

(pos. of the « gravity centre » of the peak

Strain along **K** averaged over the diffracting volume

$$\left\langle \varepsilon_{KK} \right\rangle_{\Omega} = -\frac{\mu^{(1)}}{K}$$

$$\varepsilon_{KK} = \frac{\Delta d_{hkl}}{d^{0}_{hkl}} = -\frac{\Delta \theta}{\tan \theta_0}$$

If homogeneous strain over Ω!

2nd order (centered) moment

$$\mu^{(2)} = \int_{-\infty}^{+\infty} s^2 \cdot I(s)ds$$

(« peak width »)

Variance of elastic strains along **K** over the diffracting volume

$$\left\langle \varepsilon^2_{KK} \right\rangle_{\Omega} = \frac{\mu^{(2)}}{K^2}$$

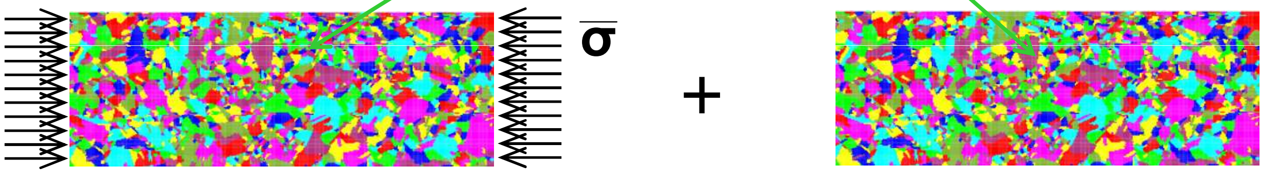
Ideal for coupling with homogenization techniques!

Interpretation of peak shifts

Stress at a given point **x**:

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{B}(\mathbf{x}) : \overline{\boldsymbol{\sigma}} + \boldsymbol{\sigma}_{res}(\mathbf{x}) \qquad \text{with} \qquad \langle \boldsymbol{\sigma}_{res} \rangle = \mathbf{0}$$

(here ≅ 2nd & 3rd order stresses)



$\overline{\boldsymbol{\sigma}}$ + $\overline{\boldsymbol{\sigma}} = \mathbf{0}$

Strain: $$\boldsymbol{\varepsilon}(\mathbf{x}) = \mathbf{S}(\mathbf{x}) : \boldsymbol{\sigma}(\mathbf{x})$$

General Formulation :

$$< \varepsilon_{KK} >_{\Omega} = \frac{\mathbf{K} \otimes \mathbf{K}}{K^2} : \langle \mathbf{S} : \mathbf{B} \rangle_{\Omega} : \overline{\boldsymbol{\sigma}} + \frac{\mathbf{K} \otimes \mathbf{K}}{K^2} : \langle \mathbf{S} : \boldsymbol{\sigma}_{\text{res}} \rangle_{\Omega}$$

The « sin² ψ law »…

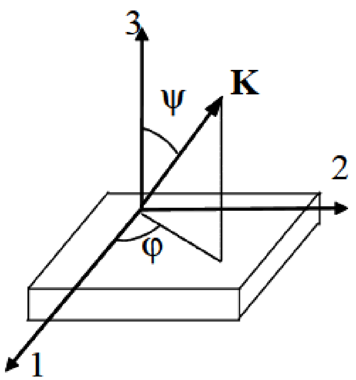
1st approximation $\boldsymbol{\sigma}_{res} = \mathbf{0}$ ➡ $< \varepsilon_{KK} >_\Omega = \frac{\mathbf{K} \otimes \mathbf{K}}{K^2} : \langle \mathbf{S} : \mathbf{B} \rangle_\Omega : \overline{\boldsymbol{\sigma}}$

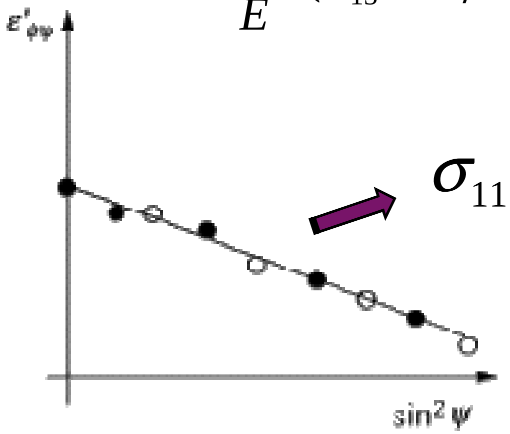
2nd approximation: isotropic elasticity…

$$< \varepsilon_{KK} >_\Omega = \frac{\mathbf{K} \otimes \mathbf{K}}{K^2} : \mathbf{S} : \overline{\boldsymbol{\sigma}}$$

➡

$$\langle \varepsilon_{KK} \rangle_\Omega = \frac{1+\nu}{E}(\sigma_{11}\cos^2\varphi + \sigma_{12}\sin 2\varphi + \sigma_{22}\sin^2\varphi - \sigma_{33})\sin^2\psi + \frac{1+\nu}{E}\sigma_{33} - \frac{\nu}{E}(\sigma_{11}+\sigma_{22}+\sigma_{33}) + \frac{1+\nu}{E}(\sigma_{13}\cos\varphi + \sigma_{23}\sin\varphi)\sin 2\psi$$





➡ $\sigma_{11}, \sigma_{22}, \sigma_{33}$

Too often applied out of hypothesis validity…

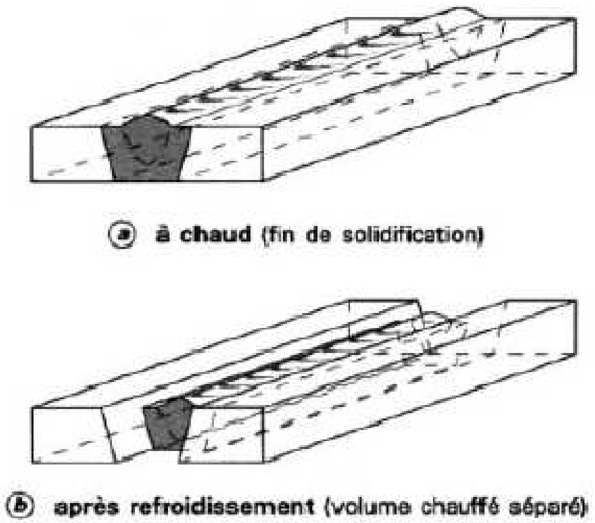
Example 1: welding

Process involving melting

Temperature gradients

Withdrawal during solidification

Residual stresses

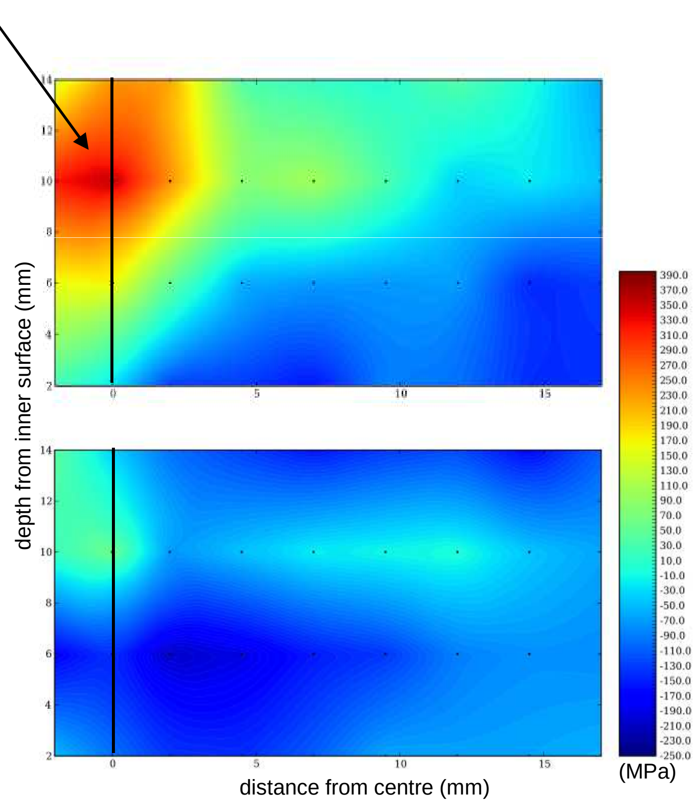


If no cohesion
(incompatibility strain field)

Determination by means of neutron diffraction

tangential stress

axial stress

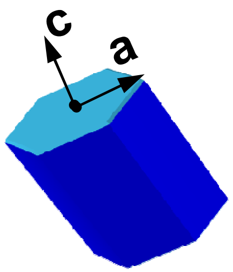


(coll. CEA/DEN)

Example 2: Intergranular stresses in Zr

SC estimates – isotropic elastic behaviour

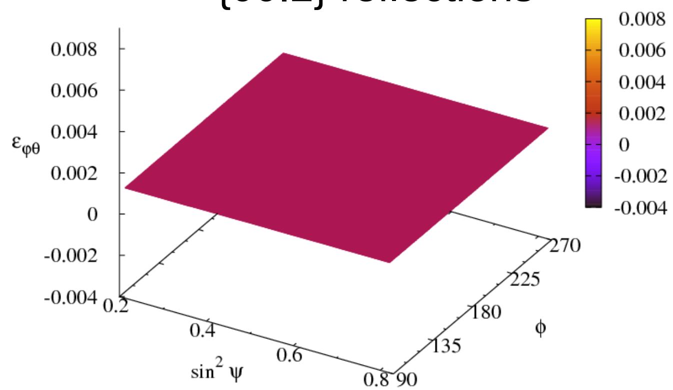
ΔT = -600°C, $\overline{\boldsymbol{\sigma}} = 0$

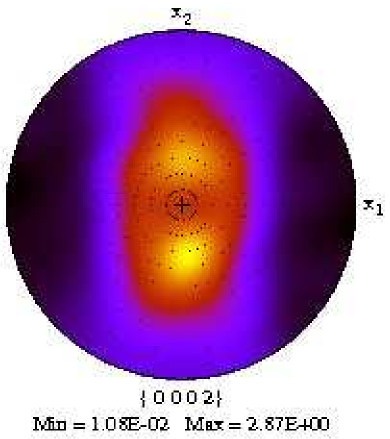


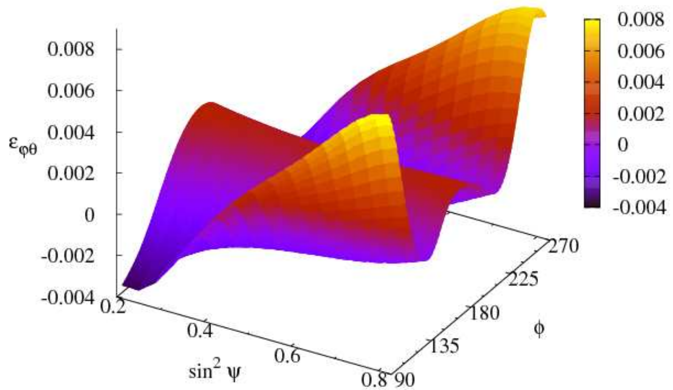
$\alpha_c \approx 2\alpha_a$

Isotropic texture

{00.2} reflections



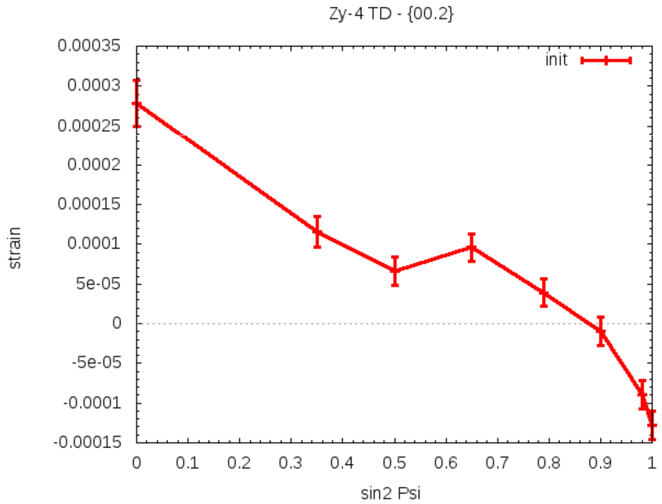


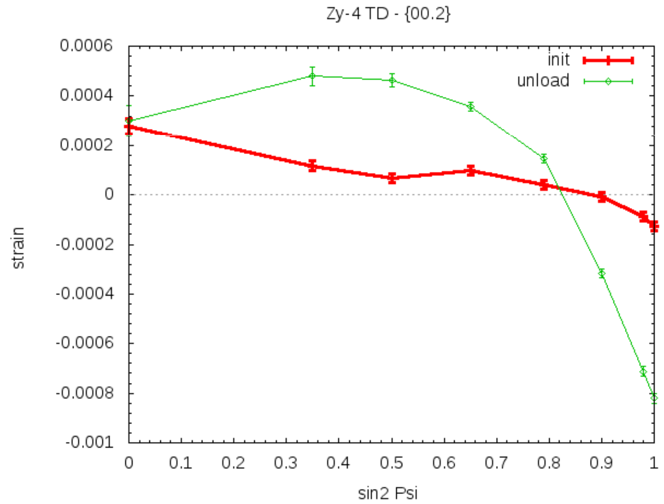


Coll. O. Castelnau (PIMM)

Example 2: Intergranular stresses in Zr

Experimental observations (neutron diffraction)





Σ = 450 MPa

(Coll. O. Castelnau (PIMM))

Importance to take microstructure into account for data analysis !!!

CONCLUSIONS

- **Various scales – various mechanisms** : need to consider physical, chemical and mechanical phenomena (i.e. **metallurgical phenomena**!)

- Stress fields are most of the time **complex** and **heterogeneous**

- Characterization of residual stresses is of fundamental and applied importances (<u>but relevant scales are not the same</u>!)

- **Various techniques** to determine residual stresses
  → Diffraction (RX & neutrons) allows to characterize intra- and intergranular heterogeneities – ideal for coupling with micromechanical modelling
  → BUT in every case: interpretating measurement results is far from being trivial
  → ~~RESIDUAL STRESS MEASUREMENT !~~

- Numerical prediction of residual stresses within components ?
  → optimization of geometries, processes, performances (*ex: LASMIS @UTT…*)

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